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Research Article

Internal benchmarking efficiency assessment in a steel company using the DEA network

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Abstract

Paper aims: This study aims to assess the network efficiency of the bar and profile rolling process in a steel manufacturing company using Network DEA (NDEA).

Originality: This research presents the first application of NDEA in combination with internal benchmarking, illustrating its feasibility in supporting significant performance improvements.

Research method: A case study was conducted in a steel plant to analyze the overall network efficiency.

Main findings: The average efficiency was 41.99%, with minimum and maximum values of 15.12% and 99.23%, respectively. Internal benchmarking revealed that the third stage of the rolling process negatively affected overall efficiency. Additionally, critical incidents influencing performance were identified, with 66.67% occurring in the fourth quarter each year.

Implications for theory and practice: Combining NDEA with internal benchmarking enables a continuous improvement framework, allowing the company to monitor and adjust operations for enhanced efficiency.

Keywords

Efficiency. Steel industry. Network data envelopment analysis (NDEA). Internal benchmarking.

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The authors have no conflict of interest to declare.

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This research did not involve experiments with human participants, nor the collection of personal or sensitive data. Therefore, ethical approval and informed consent were not required.

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1. Introduction

The steel industry plays a crucial role in global infrastructure, construction, and metal-mechanics manufacturing (Sampaio Brasil et al., 2024). Due to its strength and versatility, steel is vital in construction, machinery, transportation, rural applications, and consumer goods (Camargo et al., 2018). International trade policies, such as U.S. tariffs on steel and aluminum, can substantially affect this industry, increasing operational burdens and underscoring the need to identify efficiency constraints (Silva, 2025).

Within the steel industry, the rolling of bars and profiles generates the highest value. Measuring the efficiency of this process is therefore essential. While prior research has emphasized increasing capacity and productivity, it has often overlooked disruptions, cost implications, and scrap generation. Furthermore, production mix and water consumption—key variables in bar and profile rolling—have frequently been ignored (Sampaio Brasil et al., 2024).

Data Envelopment Analysis (DEA) is a well-established approach to evaluating productivity and efficiency in complex systems like steel manufacturing (Camanho et al., 2024). DEA compares the relative efficiency of decision-making units (DMUs) based on multiple inputs and outputs (Charnes et al., 1978) and employs benchmarking to contextualize performance (Piran et al., 2021, 2023).

Traditional DEA often functions as a "black box," providing limited insight into internal process contributions (Seth et al., 2024). Kao (2014) addressed this by proposing a foundational NDEA framework that enables the decomposition of overall efficiency into interconnected sub-processes, enhancing internal benchmarking and offering more detailed performance analysis.

Network DEA (NDEA) addresses the limitations of traditional DEA by revealing the role of sub-processes within the broader system (Färe & Grosskopf, 2000). Kao (2014) advanced this perspective by introducing a structured approach to internal benchmarking, enabling the decomposition of overall efficiency into interrelated stages. However, as Chen et al. (2013) caution, NDEA presents methodological pitfalls: while envelopment-based models are appropriate for frontier projection, they may not capture divisional efficiencies accurately, and multiplier-based models, though suitable for assessing divisions, may produce infeasible projections. These differences highlight the importance of aligning model choice with specific research objectives. NDEA has been applied across various manufacturing sectors (Chen et al., 2020; Lu et al., 2021; Omid & Zegordi, 2015) and specifically in steel production (Omid & Zegordi, 2015; Wu et al., 2017; Yang et al., 2014, 2017; Nasim et al., 2022).

Studies comparing traditional DEA and NDEA in steel production highlight the latter's ability to assess process chains and regional efficiencies. For example, Yang et al. (2017) used NDEA to assess multiple firms and regional differences, while Nasim et al. (2022) incorporated undesirable outputs using a Directional Distance Function (DDF) model. Omid & Zegordi (2015) integrated NDEA with the Analytic Hierarchy Process (AHP) to assign variable weights.

Given the limitations of traditional DEA models in capturing the complexity of internal operational structures, Network Data Envelopment Analysis (NDEA) has emerged as an alternative for assessing interconnected processes within organizations (Färe & Grosskopf, 2000; Kao, 2014). This research builds upon and extends the analytical foundations established by Piran et al. (2021), who utilized classical DEA models to evaluate internal performance, by incorporating a network-oriented approach that offers a more detailed and structured decomposition of efficiency across sub-processes. In this context, the present study advances the literature by introducing several original contributions: (1) it provides the first empirical application of NDEA using real-world data from the metal industry (unlike previous studies that applied the classical DEA approach by treating the system as a black box); (2) it enhances the understanding of how production lines and internal processes influence overall performance, thereby supporting data-driven managerial decision-making; (3) it establishes a benchmarking framework across operational sectors, enabling comparative productivity analysis among distinct units; and (4) it identifies key performance bottlenecks, proposing evidence-based strategies for continuous improvement and operational optimization.

This study addresses the following research question: What is the efficiency of the steel rolling process, considering its networked operations through the application of the Network DEA (NDEA) model using internal benchmarking? To answer this, the research evaluates the network efficiency of the bar and profile rolling process using NDEA, incorporating internal benchmarking to identify critical incidents and performance patterns. This research is the first to apply this combination and demonstrates its potential for promoting operational improvements.

The structure of this paper is as follows: Section 2 outlines the theoretical background, focusing on key NDEA studies. Section 3 details the methodology, while Section 4 presents the results, emphasizing both overall system performance and critical inefficiencies. Section 5 discusses theoretical contributions, followed by the conclusions in Section 6.

2. Theoretical background

Adopting a development-oriented approach to NDEA, Zhang & Zhang (2025) evaluated the energy supply system in China. The study incorporated components such as electricity production, electricity sales, and the generation of both fossil and non-fossil energy. The findings indicate that inefficiency primarily occurs in non-fossil energy substage. Additionally, the study highlights the importance of incorporating energy-related variables into the development of NDEA models (Zhang & Zhang, 2025).

NDEA models can be applied across various sectors, including production and finance. In the case of a two-stage NDEA, the first stage should incorporate inputs and intermediate outputs, while the second stage should include additional inputs, intermediate outputs, and final outputs (Zibaei Vishghaei et al., 2024). An example of this application is found in the oil production sector, where two stages were assessed: the first evaluated the capacity to convert resources into actual production, and the second measured how production from the first stage was transformed into profit (Jung et al., 2025).

Nevertheless, studies examining efficiency trends over time in the steel industry remain scarce. Yang et al. (2014) analyzed pig iron production as an intermediate stage and crude steel as the final product using a three-stage NDEA model. The model employed capital investment and total labor as inputs in the first stage, pig iron as the second-stage output and third-stage input, and crude and finished steel as final outputs.

In another study, Yang et al. (2017) compared DEA and NDEA in casting and billet production. Inputs such as casting time, alloy content, steel scrap, and electricity were used in the first stage, with billets produced and later serving as inputs for the third stage. The final output was qualified steel, derived after additional gas and electricity consumption.

Wu et al. (2017) integrated sustainability into NDEA by including environmental variables. The three-stage model used energy, freshwater, and iron ore in the first stage. Outputs like pig iron and crude steel were transferred between stages, along with wastewater, environmental staff, and treatment costs. Final outputs included wastewater disposal and reusable water.

Yang et al. (2017) also analyzed technical efficiency across regions using data from 26 firms. Inputs included fixed asset investment and workforce size, with pig iron produced in the second stage and crude and finished steel in the final stage.

Applications of NDEA extend beyond steel. Omid & Zegordi (2015) assessed a Chinese carpet firm using 15 DMUs. Their three-stage model used raw materials, wages, and expenses to produce perfect carpets, which were then used to generate internal and external sales profits.

Lu et al. (2021) evaluated eight machinery firms in Taiwan using a similar model. Barat et al. (2019) applied a three-stage network to 20 petrochemical companies, and Nasim et al. (2022) extended NDEA to a four-stage model assessing 42 cement firms. Song et al. (2020) examined airline efficiency across national markets and Khoveyni & Eslami (2022) analyzed a manufacturing supply network consisting of 20 supply chains.

3. Methodological procedures

This study adopts a case-based research approach, which is particularly appropriate for investigating complex social phenomena (Yin, 2005). As an empirical method, the case study enables the examination of one or more units of analysis, allowing for a deeper understanding of the phenomenon under investigation (Miguel, 2007; Eisenhardt & Graebner, 2007).

Specifically, we conducted a longitudinal case study with a quantitative focus, appropriate for single-case designs, as it enhances internal validity (Piran et al., 2020; Sampaio Brasil et al., 2024). The study was structured in two main stages: (i) defining the case and the NDEA model, and (ii) data collection and analysis. The details of the method, which includes the two main stages described above, are presented in Figure 1.

Given that the case was developed in the largest steel company in Brazil, the stages outlined in Figure 1 provide a replicable framework for applying the NDEA model to evaluate efficiency in steel industries.

3.1. Definition of the case and the NDEA model

The case involves a steel company responsible for both domestic and international operations in scrap metal recycling and steel production. Its primary clients are from the construction, automotive, and agricultural sectors. The company operates 40 plants, employs over 30,000 people across 12 countries, and reported a net revenue of USD 13 billion in 2024.

The company operates under a semi-integrated model, organized into production and support sectors. Production includes the melt shop, rolling mills, drawing mills, nail factory, and finished products. Support functions encompass maintenance, utilities, administration, and human resources.

This study focuses on the rolling mill process, which manufactures reinforcing steel and profiles used in the construction and metalworking industries. In this process, billets are fed into the rolling mill via a roller path to achieve the desired shape. Once shaped, the billet becomes a "piece," which proceeds to the cutting stage, still

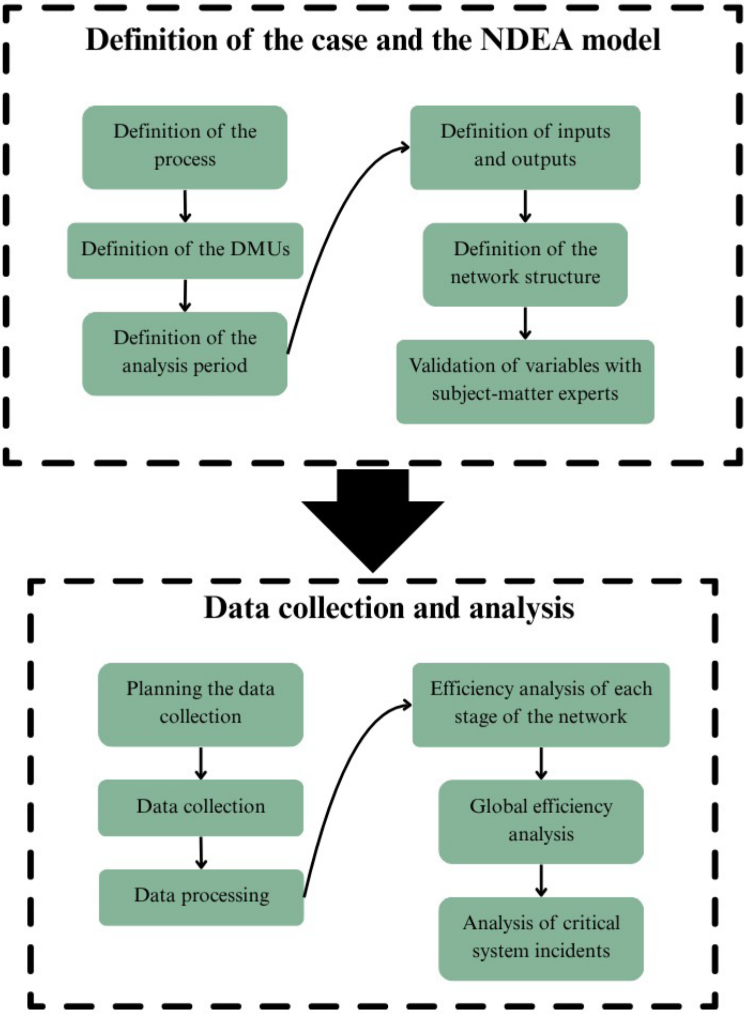

Figure 1. Research Procedure Stages.

on the roller path. After cutting, the pieces are moved to the cooling bed, then bundled, weighed, identified, and dispatched to storage. The entire process is automated and managed from control rooms.

Following the definition of the production process, a team of experts was assembled to support the study and contribute to the development of the DEA model (Table 1). The selected experts included a mechanical

Table 1. Group of company experts.

Position	Length of time with the company	Education
Mechanical engineer (specializing in rolling mill maintenance)	20 years	Degree in mechanical engineering; Specialization in hydraulics, steel processes, and maintenance engineering.
Process engineer	30 years	Degree in mechanical engineering; Specialization in rolling mill processes, work safety, and business management.
Production engineer	15 years	Degree in Production Engineering; Specialization in People Management.
Operator in charge of central control	28 years	Technical level; Specialization courses in rolling processes. Benchmarking in other companies in the group.
Automation technical assistant	18 years	Degree in electrical engineering; Postgraduate degree in industrial automation.

engineer specializing in rolling mill maintenance, a process engineer, a production engineer, a central control operator, and a technical assistant in automation.

The efficiency levels were estimated under the assumption of constant returns to scale (CRS), given the comparable scale size among DMUs. This assumption aligns with the internal benchmarking context, where the unit of analysis is evaluated against itself over time (e.g., Piran et al., 2020, 2021). An output-oriented CRS model was applied using a series structure, where Zp represents the intermediate products, Xj the inputs, and Y_r the outputs. The model follows the generalized formulation presented by Kao and Hwang (2008), defined by Equations 1 to 9.

$$E_k = max \sum_{r=1}^{s} u_r \, .Y_{rk} \tag{1}$$

Subject to:

$$\sum_{i=1}^{m} v_i \cdot X_{ik} = 1 \tag{2}$$

$$\sum_{r=1}^{s} u_r \cdot Y_{rj} - \sum_{i=1}^{m} v_i \cdot X_{ij} \leq 0,\ j = 1,\ldots,n \tag{3}$$

$$\sum_{p=1}^{t} w_p^{(1)} Z_{pj}^{(1)} - \sum_{i=1}^{m} v_i X_{ij} \leq 0, j = 1,\ldots,n \tag{4}$$

$$\sum_{p=1}^{t} w_p^{(t)} Z_{pj}^{(t)} - \sum_{p=1}^{t} w_p^{(t-1)} Z_{pj}^{(t-1)} \leq 0, \tag{5}$$

$$t=2,\ldots,h-1, j=1, \ldots,n \tag{6}$$

$$\sum_{r=1}^{s} u_r \cdot Y_{rj} - \sum_{p=1}^{q} w_p^{(t)} Z_{pj}^{(t)} \leq 0, j = 1,\ldots,n \tag{7}$$

$$u_r, v_i, w_p^{(t)} \geq \varepsilon \tag{8}$$

$$r = 1, \ldots, \text{s},\ i = 1, \ldots, \text{m},\ p = 1, \ldots \text{q}, t = 1, \ldots, h-1 \tag{9}$$

Where:
u_r = Weight applied to output r
v_i = Weight applied to input i
w_p = Weight applied to intermediate variable p
X_{ij} = Quantity of input i in DMU j
Y_{rj} = Quantity of output r in DMU . j .
Z_{pj} = Quantity of Variable p in DMU j
Y_{rk} = Quantity of output r in the DMU under analysis
X_{ik} = Quantity of input i in the DMU under analysis
r = Index of outputs
i = Index of inputs
k = Index of the decision-making unit (DMU) under evaluation
p = Index of intermediate variables
. t . = Index of the stage in multi-stage process models
j = Quantity of DMUs
m = Quantity of inputs
= Quantity of outputs
q = Quantity of intermediate variables
h = Quantity of stages in the model

Intermediate products from stage t serve as outputs for stage t and inputs for stage $t+1$. In the last process, h, the intermediate products correspond to the outputs of the system. Constraints in Equation 3 apply to the entire system, while those in Equations 4, 5 and 7 apply to individual stages. The constraint system is redundant and could be omitted. Therefore, the number of constraints is equal to the number of DMUs multiplied by the number of processes in the system. Thus, a DMU is only considered efficient if all its processes are efficient. Therefore, the efficiency of each process will be given by Equations 10-12 where u_r*, v_i* and $wp(t)*$ are the optimal multipliers (Kao, 2009).

$$E_k^{(1)} = \frac{\sum_{p=1}^{q} w_p^{(1)} z_{pk}^{(1)}}{\sum_{i=1}^{m} v_i^* x_{ik}} \tag{10}$$

$$E_k^{(t)} = \frac{\sum_{p=1}^{q} w_p^{(1)} z_{pk}^{(1)}}{\sum_{i=1}^{m} w_p^{(t-1)} z_{pk}^{(t-1)}}, t = 2, \ldots, h-1 \tag{11}$$

$$E_k^{(h)} = \frac{\sum_{r=1}^{s} u_r^* y_{rk}}{\sum_{p=1}^{q} w_p^{(h-1)} z_{pk}^{(h-1)}} \tag{12}$$

DMUs were defined based on previous NDEA applications in the steel industry (Yang et al., 2014; Wu et al., 2017; Yang et al., 2017; Barat et al., 2019; Chen et al., 2020; Song et al., 2020; Lu et al., 2021; Nasim et al., 2022; Khoveyni & Eslami, 2022). Based on literature and expert analysis, product families from the rebar and profile rolling lines were selected as DMUs. Data was organized quarterly, allowing for the inclusion of product families that were inactive during certain months. The analysis period spans January 2020 to December 2023, encompassing four years of production and input data. This phase allowed for comprehensive data acquisition.

The network, shown in Figure 2, comprises 14 variables across three stages and includes data from 16 DMUs.

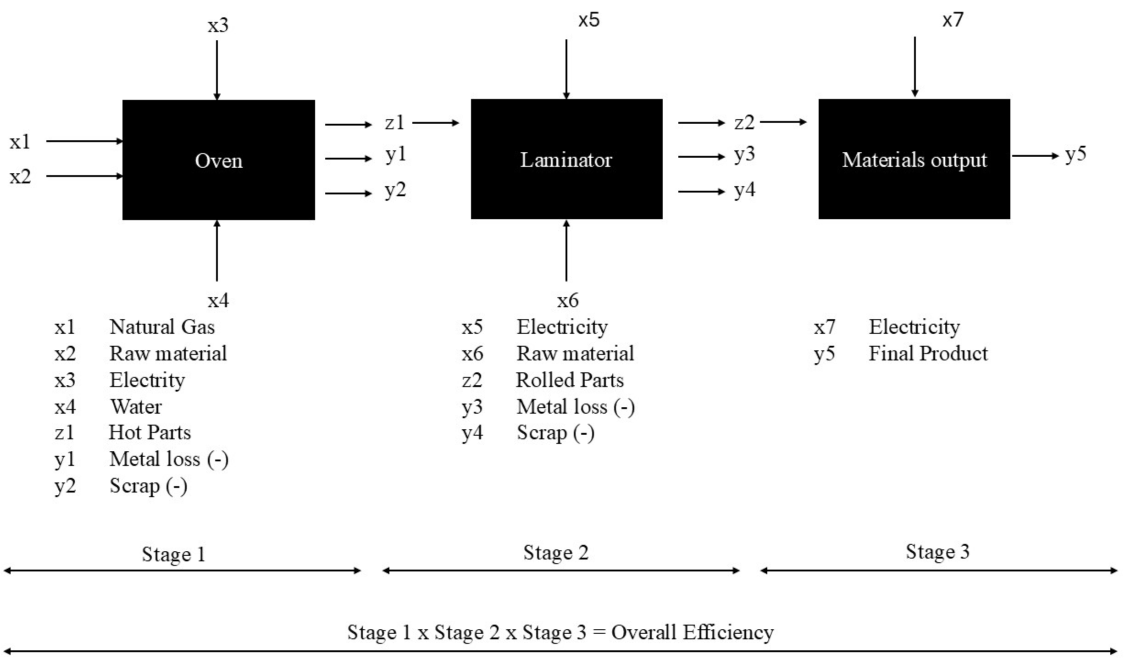

Figure 2. NDEA Network.

After defining the variables, the model was reviewed by a group of specialists to assess data availability. The specialists confirmed that the required data could be retrieved from various company systems. Accordingly, data related to the operation under analysis were obtained from the following systems: (i) MES (Manufacturing Execution System) EPS (eProductivity Software): a system used to monitor and control production activities on the shop floor; (ii) SIGE (Energy Management System): enables the collection of data related to electricity consumption; (iii) Utility Supervisory; and, (iv) SAP (Systems Applications and Products in Data Processing): provides comprehensive control over various business processes, supporting integrated enterprise management.

3.2. NDEA data collection and analysis

Table 2 summarizes the data collection methods defined for each variable in the model, along with the respective systems from which the data were obtained.

Table 2. Data collection planning.

Variable	Form of calculation	System to be collected
Steel billet	The sum of the number of tons used to produce a given product family.	MES EPS
Natural gas	The sum of the volume of natural gas used to produce a given product family.	SIGE
Electricity	The sum of the amount of electricity used to produce a given product family.	SIGE
Water	The sum of the volume of raw water used to produce a given product family.	Utility Supervisory
Hot parts	The sum of the number of tons used to produce a given product family, excluding possible scrap and metal loss.	MES EPS
Product	The sum of the good parts produced in each product family.	MES EPS
Scrap (-)	Sum, in tons, of the parts lost due to interruption.	SAP
Metal loss (-)	Sum, in tons, of the raw materials lost due to discontinuations inherent in the production of each product family.	SAP

Table 3 presents the descriptive statistics for the variables, including minimum, maximum, mean, and standard deviation. It also lists previous studies that have addressed each variable.

The primary raw material used in the production of profiles and rebars is the billet, which undergoes a heating process before being rolled to produce finished steel and its subsequent products. The billet serves as an input in the first stage of the process (Omid & Zegordi, 2015; Yang et al., 2017, Nasim et al., 2022).

Water is a shared input across subprocesses and is essential for cooling both the furnace instrumentation and the raw material as it passes through the rolling stage (Nasim et al., 2022; Wu et al., 2017).

Electricity is another key input, utilized throughout all stages of production and across subprocesses (; Wu et al., 2017; Yang et al., 2017; Nasim et al., 2022). Natural gas is used to heat the furnaces and is categorized as an initial input in the first stage (Yang et al., 2017).

Following the heating stage, the output consists of heated parts that must reach a standardized temperature before proceeding to the rolling process. These hot parts then become an input in the second stage (Nasim et al., 2022; Wu et al., 2017).

The data analysis first evaluates the efficiency of the three stages in the serial network using Network Data Envelopment Analysis (NDEA), with a focus on internal benchmarking and overall network performance. The analysis also includes the assessment of critical incidents in the system.

The combination of the critical incident technique with internal benchmarking is useful for examining the impact of management actions or interventions over time (Piran et al., 2023).

Critical incidents are classified as either external (ECI) or internal (ICI). ECIs refer to non-controllable events that influence system outcomes, while ICIs are controllable events or managerial actions aimed at improving efficiency (Piran et al., 2021).

4. Results analysis

4.1. Overall system efficiency

The model was applied to calculate the efficiencies of the DMUs within the network. The overall average efficiency was 41.99%, with a minimum of 15.12% and a standard deviation of 0.1990. Table 4 presents the

Table 3. Descriptive statistics of the variables.

Type	Variable	Statistic	Value	Previous studies
Input	Natural gas (x_1)	Minimum	42,020.60	Yang et al. (2017)
		Maximum	85,285.79	
		Mean	64,267.71	
		Standard deviation	12,936.08	
Input	Raw materials (x_2)	Minimum	419.54	Omid & Zegordi (2015), Nasim et al. (2022), Yang et al. (2017)
		Maximum	1,908.20	
		Mean	1,245.78	
		Standard deviation	366.47	
Input	Electricity (x_3)	Minimum	2,236.09	Nasim et al. (2022), Wu et al. (2017), Yang et al. (2017)
		Maximum	4,016.85	
		Mean	3,378.93	
		Standard deviation	556.16	
Input	Water (x_4)	Minimum	7,008.16	Nasim et al. (2022), Wu et al. (2017)
		Maximum	11,326.74	
		Mean	8,748.45	
		Standard deviation	1,282.34	
Input	Electricity (x_5)	Minimum	97,269.97	Nasim et al. (2022), Wu et al. (2017), Yang et al. (2017)
		Maximum	174,732.94	
		Mean	146,983.47	
		Standard deviation	24,193.15	
Input	Water (x_6)	Minimum	13,015.15	Nasim et al. (2022), Wu et al. (2017)
		Maximum	21,035.37	
		Mean	16,247.13	
		Standard deviation	2,381.50	
Input	Electricity (x_7)	Minimum	12,298.50	Nasim et al. (2022), Wu et al. (2017), Yang et al. (2017)
		Maximum	22,092.67	
		Mean	18,584.12	
		Standard deviation	3,058.90	
Intermediate variable	Hot parts (z_1)	Minimum	418.90	Nasim et al. (2022), Wu et al. (2017)
		Maximum	1,905.28	
		Mean	1,243.88	
		Standard deviation	365.91	
Intermediate variable	Rolled parts (z_2)	Minimum	413.77	Nasim et al. (2022), Wu et al. (2017)
		Maximum	1,881.93	
		Mean	1,228.64	
		Standard deviation	361.43	
Output	Metal loss (y_1)	Minimum	0.06	Yang et al. (2017)
		Maximum	0.29	
		Mean	0.19	
		Standard deviation	0.06	
Output	Scrap (y_2)	Minimum	0.58	Yang et al. (2017)
		Maximum	2.63	
		Mean	1.71	
		Standard deviation	0.50	
Output	Metal loss (y_3)	Minimum	5.77	Yang et al. (2017)
		Maximum	26.26	
		Mean	17.15	
		Standard deviation	5.04	
Output	Scrap ($_4$)	Minimum	1.28	Yang et al. (2017)
		Maximum	5.84	
		Mean	3.81	
		Standard deviation	1.12	
Output	Final product (y_5)	Minimum	406.71	Omid & Zegordi (2015), Wu et al. (2017)
		Maximum	1,849.84	
		Mean	1,207.68	
		Standard deviation	355.26	

Table 4. Overall efficiencies.

Reference	DMU	Overall Efficiency	Efficiency Stage 1	Efficiency Stage 2	Efficiency Stage 3
1st quarter 2020	DMU 01	52.78%	82.89%	87.36%	72.89%
2nd quarter 2020	DMU 02	50.22%	88.13%	70.48%	80.85%
3rd quarter 2020	DMU 03	39.01%	79.35%	66.77%	73.64%
4th quarter 2020	DMU 04	30.53%	80.93%	64.61%	58.39%
1st quarter 2021	DMU 05	55.76%	88.23%	90.16%	70.10%
2nd quarter 2021	DMU 06	99.23%	100%	99.23%	100%
3rd quarter 2021	DMU 07	38.89%	79.16%	68.84%	71.36%
4th quarter 2021	DMU 08	28.44%	74.22%	72.03%	53.20%
1st quarter 2022	DMU 09	43.79%	76.20%	79.41%	72.37%
2nd quarter 2022	DMU 10	43.88%	74.79%	68.67%	85.43%
3rd quarter 2022	DMU 11	17.77%	68.05%	47.80%	54.62%
4th quarter 2022	DMU 12	15.12%	73.62%	94.97%	21.63%
1st quarter 2023	DMU 13	56.61%	77.74%	100%	72.81%
2nd quarter 2023	DMU 14	32.76%	70.62%	70.06%	66.20%
3rd quarter 2023	DMU 15	43.52%	75.14%	67.83%	85.39%
4th quarter 2023	DMU 16	23.50%	75.74%	56.57%	54.85%
Mean		41.99%	79.05%	75.30%	68.36%
Standard deviation		0.1990	0.0784	0.1517	0.1775
Minimum		15.12%	68.05%	47.80%	21.63%
Maximum		99.23%	100%	100%	100%

sixteen quarters analyzed, representing the sixteen DMUs in the network and covering a continuous 48-month period from January 2020 to December 2023.

As shown in Table 4, higher DMU performance within the network corresponds to a higher overall efficiency score. DMU 06 (2nd quarter of 2021) demonstrated the highest efficiency. In contrast, DMUs 12 (Q4 2022), 11 (Q3 2022), 16 (Q4 2023), and 8 (Q4 2021) recorded the lowest efficiency scores during the analyzed period. In DMUs 8, 12, and 16, efficiency was significantly affected by a reduction in production volumes (y5), primarily due to regular corrective and preventive maintenance stoppages across all equipment. For DMU 11, the reduction in production volume (y5) was attributed to elevated inventory levels during the same period.

To support the analysis of network efficiency, Figure 3 provides a graphical representation of the overall efficiency values from Table 4. This visualization helps to identify efficiency trends over time.

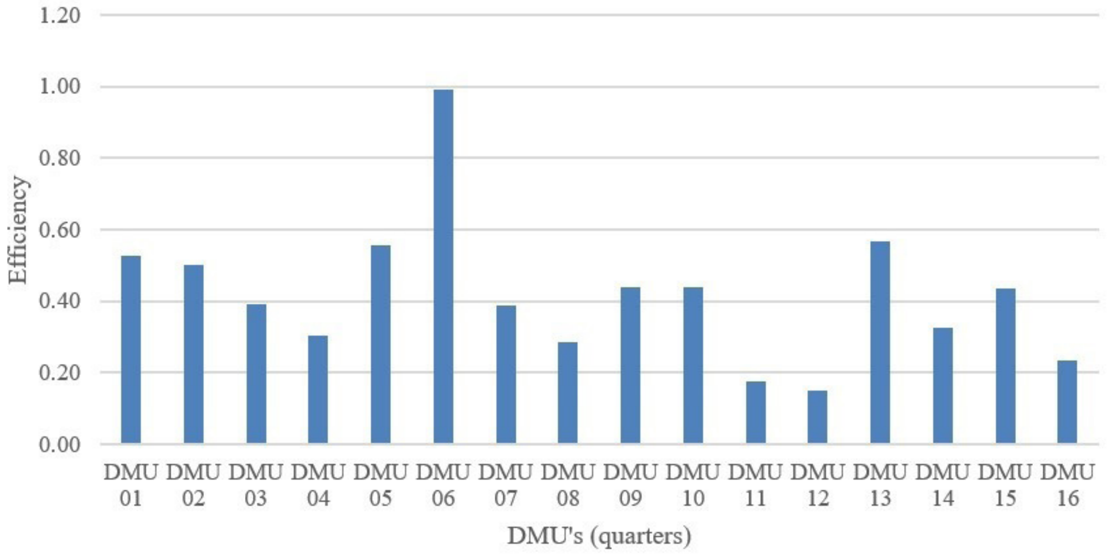

Figure 3. Global efficiencies.

Understanding the contribution of each stage to the overall efficiency is essential for identifying key improvement areas or process bottlenecks. Figure 4 illustrates how variations in the efficiency of individual stages influence the overall efficiency score. These fluctuations emphasize the need for a detailed assessment to identify specific causes and to guide management in developing targeted strategies.

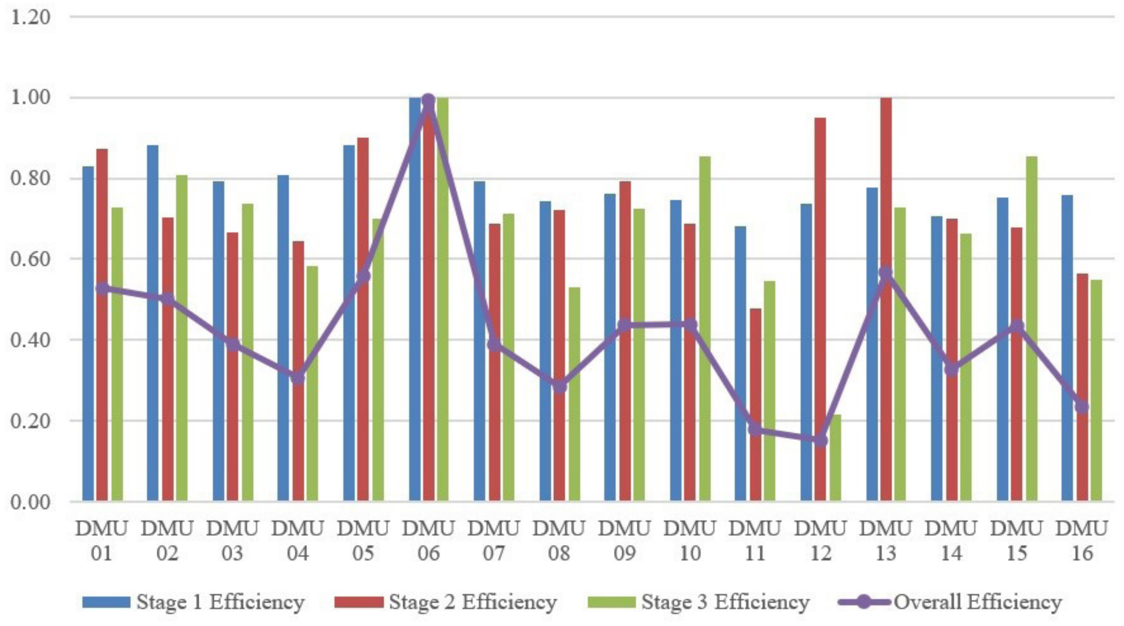

Figure 4. Integrated analysis of efficiencies.

To examine average efficiencies by quarter, Table 5 was created. The data reveal that the lowest averages occurred in the fourth quarter of each year, with a mean efficiency of 24.40%. To broaden the analysis, annual mean efficiencies were also calculated. The results indicate variability across the years analyzed, with the lowest average recorded in 2021 (30.14%) and the highest in 2020 (55.58%).

Table 5. Quarterly and year efficiency means.

Quarter	Mean efficiency	Year	Mean efficiency
1st quarters	52.23%	2020	43.13%
2nd quarters	56.52%	2021	55.58%
3rd quarters	34.80%	2022	30.14%
4th quarters	24.40%	2023	39.10%

Understanding how each stage contributes to overall efficiency helps identify key areas for improvement or potential bottlenecks in the process, enabling more effective intervention. Among the sixteen DMUs evaluated, 56.25% showed a negative contribution from the third stage to the overall efficiency, highlighting the need to monitor the material output from the rolling mill. Meanwhile, the second stage had a negative contribution in 43.75% of the DMUs.

4.2. Analysis of critical incidents

The analysis began with the validation of efficiency behavior, followed by the identification of critical incidents. A total of five internal and one external critical incident were identified, as illustrated in Figure 5.

It is noteworthy that DMU 06 was the only case with an external critical incident. The events influencing overall efficiency were identified in collaboration with an expert group, using company records such as logbooks,

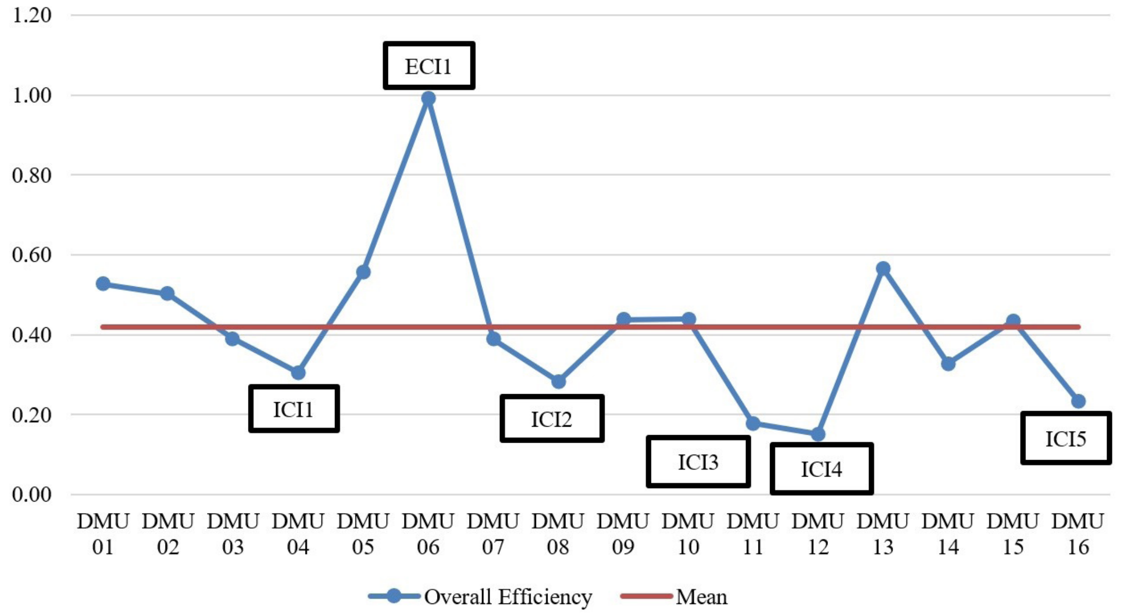

Figure 5. Identification of critical incidents.

scheduling emails, and disruption reports. Subsequently, a survey of efficiency history was conducted, which included the time preceding each event, a detailed description, and the impact of each incident, as summarized in Table 6.

Table 6. Summary of critical incidents.

Critical incident	DMU	Quarter	Description of critical incident	Overall efficiency effect
ICI 1	DMU 04	4th quarter 2020	Maintenance stoppage from December 2 to 30. No production occurred during this period.	Efficiency decreased by approximately 20%.
ICI 2	DMU 08	4th quarter 2021	Maintenance stoppage from December 1 to 20. The restart was problematic, with several process disruptions.	Efficiency decreased by approximately 45%.
ICI 3	DMU 11	3rd quarter 2022	Rolling mill stoppage (2nd stage) for 120 hours. Over 20 tons of parts lost; 400 tons below the scheduled production volume.	Efficiency decreased by approximately 25%.
ICI 4	DMU 12	4th quarter 2022	Cooling chute stoppage (3rd stage) for 96 hours.	Efficiency decreased by approximately 5%.
ICI 5	DMU 16	4th quarter 2023	Maintenance stoppage from December 1 to 29. No production occurred during this period.	Efficiency decreased by approximately 18%.
ECI 1	DMU 06	2nd quarter 2021	Following the onset of the pandemic, the plant achieved record production due to supply shortages across the industry.	Efficiency increased by 50% (compared to the average of prior quarters). Production volume was 48% higher than the average of the previous five quarters.

Maintenance shutdowns are typically scheduled in the fourth quarter, particularly in December, as observed in ICI 1, 2, and 5. In ICI 1 and 5, maintenance lasted approximately 30 days, during which no material was produced. Although the shutdown in ICI 2 was shorter, the restart did not proceed as planned, adversely affecting key performance indicators, especially efficiency.

ICI 3 and ICI 4 were triggered by corrective maintenance, initiated upon detection of equipment faults during regular operations. These interruptions involved component replacements, emergency repairs, or other necessary actions to restore equipment functionality.

5. Discussion of results

This study advances theoretical knowledge by contributing to the understanding and application of Network Data Envelopment Analysis (NDEA). Among its various advantages, a key strength of NDEA lies in its capacity to provide an internal visualization of each Decision-Making Unit (DMU), enabling detailed analysis of sub-processes. While traditional DEA evaluates the efficiency of a DMU using only inputs and outputs, NDEA extends this analysis by assessing the efficiency of each sub-process within the system (Omid & Zegordi, 2015).

Prior studies suggest that NDEA has primarily been applied in empirical contexts, emphasizing its relevance to real-world cases (Amir & Reza, 2022; Yang et al., 2014, 2017). However, the limited number of practical studies has constrained comprehensive evaluations of its benefits (Kao, 2014; Ratner et al., 2023).

This research presents all efficiency indices, both for individual stages and overall system performance. This approach enables the identification of relationships and sub-processes that contribute to reduced overall efficiency. Some studies report both overall and stage-specific efficiency indices (Lu et al., 2021; Wu et al., 2017), while others focus solely on stage-level indicators (Amir & Reza, 2022).

The contribution of NDEA combined with internal benchmarking lies primarily in assessing and improving a company's efficiency. By applying NDEA alongside internal benchmarking, firms can identify specific areas where performance falls short of their maximum efficiency potential, based on comparisons with their own best practices or historical performance. It also supports a continuous improvement strategy through regular performance monitoring and strategic adjustments. To the best of our knowledge, this is the first study to apply NDEA for internal benchmarking, which may encourage further practical applications of the method.

The study has practical implications for the company under analysis by validating the usefulness of combining NDEA with internal benchmarking to assess process efficiency. Process experts acknowledged that the NDEA-derived efficiency scores offer a simplified framework for understanding the performance of operational units and sub-processes.

An analysis of the results indicates several reasons for the poor performance observed in certain DMUs, which experienced a significant decline in production volumes. For DMUs 8, 12, and 16, the main factor was the occurrence of routine corrective and preventive maintenance across all equipment. In the case of DMU 11, the underperformance was due to a high inventory level during the period. Understanding these practical causes of inefficiency enables the implementation of improvement actions, such as enhancing predictive maintenance practices.

Additionally, the use of critical incident analysis proved valuable. This method helps identify vulnerable areas within the process, allowing for targeted risk mitigation strategies. It also facilitates the recognition of recurring patterns that may contribute to persistent inefficiencies. The study is especially relevant to industrial sectors such as steel manufacturing, where operational complexity necessitates robust tools to evaluate efficiency and support decision-making.

6. Conclusion

This study aimed to evaluate the network efficiency of the steel rolling process in a bar and section manufacturing company by applying NDEA in combination with internal benchmarking. The case study was conducted in a Brazilian steel company specializing in the production of reinforcing bars and sections. The application of NDEA proved to be an effective tool for analyzing the organization's efficiency performance.

From a theoretical standpoint, this research contributes to NDEA literature by applying the method in a real-world context and by incorporating internal benchmarking and critical incident analysis. Practically, the findings highlight the importance of understanding sub-process efficiencies to improve overall system performance. Furthermore, the results confirm that internal benchmarking using NDEA can drive meaningful process improvements.

The study also emphasizes the importance of critical assessments of NDEA applications in manufacturing settings, which are crucial to maximizing the method's relevance and applicability. Analyzing sub-process efficiency in relation to overall performance enables the identification of key improvement areas, thereby enhancing system effectiveness. Moreover, the analysis of critical incidents provided insight into challenges and patterns that may inform preventive and corrective measures.

Despite these contributions, the study presents certain limitations. The NDEA model did not allow for inter-plant comparisons due to confidentiality constraints, requiring adjustments to the model and the adoption of internal benchmarking. Moreover, the analysis was conducted on a quarterly basis, which may limit the generalizability of the findings. The study also did not explore in depth the bottlenecks that affect system efficiency. Future research is encouraged to examine these dynamics more thoroughly.

This research lays the groundwork for future investigations on efficiency behavior by integrating NDEA with critical incident analysis. Future studies could expand on this work by incorporating external benchmarking, assessing targets and slack, and exploring two-dimensional representations of NDEA models within both internal and external benchmarking contexts.

Data availability

Research data is not available.

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