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Research Article

A novel methodology to obtain optimal economic indicators based on the Argentinean production chain under uncertainty

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Abstract

Paper aims: This novel methodology obtains the optimal economic evaluation of emissions (carbon price) under uncertainty (Fuzzy Decision Making), and hierarchical variation (Analytic Hierarchy Process) within the Argentine production chain (Life Cycle Analysis with Supply and Demand Side Management), obtaining a novel model of market equilibrium.

Originality: 1) a novel optimal economic (marginal) evaluation index called Generic Camargo Intrinsic Cost, 2) optimal graphical attribute efficiency points, regions and boundaries and their optimal economic evaluation and 3) a Computable General Equilibrium Model with fundamental uncertainty.

Research method: The theoretical, practical and economic contribution and results (mathematical and graphical analysis) of this novel methodology and tools are developed, generalised and analysed.

Main findings: All three of the above original contributions have been analysed using the above research method, with excellent and promising results.

Implications for theory and practice: The optimal economic evaluation (externality) of the Argentinean production chain under uncertainty is obtained.

Keywords

Argentinean Production Chain (APC). Fuzzy Decision Making (FDM). Analytic Hierarchy Process (AHP). Particle Swarm Optimisation (PSO). Life Cycle Analysis (LCA). Supply Side Management (SSM). Demand Side Management (DSM) optimisation. Computable General Equilibrium Model with fundamental uncertainty.

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1. Introduction

The ongoing challenge of addressing the continued growth of emissions and their associated environmental impacts, such as global warming, underscores the critical importance of considering both cases: improving energy efficiency in the production chain. This is crucial because of its economic, technical, environmental and social implications. By reducing the cost of energy consumption, significant savings can be made and economic competitiveness improved, while at the same time mitigating climate change and improving energy security. Various approaches are being used to address this challenge, from the development of more efficient technologies to the adoption of government policies, public education and ongoing research into novel solutions. Improving energy efficiency is essential to ensure a sustainable and secure future for future generations.

The current focus is therefore on two cases of energy efficiency management, covering both the supply and the demand side. Supply Side Management (SSM) strategies involve optimising energy production and distribution to meet demand efficiently and sustainably. On the other hand, Demand Side Management (DSM) strategies aim at changing consumption patterns at different levels of use, minimising the use of appliances and optimising their operation. DSM includes strategies such as time-of-use pricing, demand response programmes, energy efficiency initiatives and load management technologies that influence consumer electricity use patterns, optimise energy consumption, reduce peak demand and improve grid reliability and efficiency. These strategies focus on improving the performance and reliability of energy generation sources, upgrading transmission and distribution infrastructure, promoting the use of renewable energy and integrating advanced technologies for grid optimisation and flexibility. The aim of both supply and demand management is to ensure a reliable, affordable and environmentally sustainable energy supply to meet current and future needs. In the context of Industry 4.0, optimisation refers to the process of improving and maximising the performance, productivity and efficiency of both supply and demand management strategies. This holistic approach aims to leverage technological advances and data-driven insights (subject to uncertainty) to improve energy management practices (Camargo, 2023; Rodríguez et al., 2023; Causil & Morais, 2023).

In this context, where energy efficiency technologies and technical, economic and environmental optimisation play a crucial role, there is a close relationship with carbon pricing, which acts as a Pigouvian tax mechanism to address the positive (positive effects) and negative (negative effects) externalities associated with property rights in CO_2 emissions. Carbon pricing provides economic incentives or penalties (depending on the externality produced) for companies to adopt cleaner and more energy-efficient technologies and practices, thereby stimulating investment in innovation and enhancing long-term competitiveness, while contributing to climate change mitigation and promoting a more sustainable economy. They are determined by the laws of supply and demand, based on the trading of property rights, and depend on the caps, floors and tiers of the sectors involved. In the current state of the art, these data are often obtained through statistical analysis (subject to uncertainty) and complex mathematical modelling in operations research (see Table 1). Uncertainty refers to the lack of knowledge about the impact of one action on another, and this can be none (deterministic models), partial (probabilistic or stochastic models) and complete or fundamental (fuzzy models). In the context of the economic evaluation of Pigouvian carbon taxes, the type of uncertainty plays an important role in understanding the carbon price, as it affects the tools that can be applied and the results obtained (see Table 1). This is even more critical when studying variables that are not linked to a market governed by the laws of supply and demand, i.e. non-monetary indices (Fuentes-Morales et al., 2020; Hassan, 2021; Wu et al., 2022; Camargo, 2019, 2021, 2022a, b, 2023; Rodríguez et al., 2023; Causil & Morais, 2023).

Table 1 provides a summary of current proposals for economic evaluations with carbon pricing and CO_2 emissions, including: country, uncertainty, modelling, indices studied and comments. They approach the solution of single-objective and multi-objective problems by applying the techniques available in the state of the art with difficulty (problems of metric compatibility between indices). This implies the use of subjective linear weights that balance the units in the construction of the objective function to be optimised and, in turn, weight these indices, leading to solutions that may be suboptimal. In addition, these proposals represent complex methods that require large amounts of data with uncertainty (none, partial and fundamental) and the use of Artificial Intelligence (AI) with supervised learning (machine learning), and most of them do not take into account the presence of fundamental uncertainty, but only partial uncertainty.

Artificial intelligence for problem solving and optimisation refers to a set of techniques designed to find effective and efficient solutions to complex problems across multiple disciplines. AI ranges from heuristic algorithms, which explore promising solutions without guaranteeing optimality, to algorithmic search techniques, which aim to find optimal or suboptimal solutions within a defined search space. In addition, AI includes diverse approaches such as heuristic search, informed search, machine learning (e.g. neural networks), and metaheuristic optimisation (particle swarm optimisation), which are applied in areas such as planning, logistics, system optimisation, and economics to solve hard problems and improve decision making. Other papers present techniques such as computable general equilibrium models, which use real economic data to estimate how an economy might respond to changes in policy, technology or other external factors that are subject to uncertainty (Camargo et al., 2019; Camargo, 2019, 2021, 2022a, b, 2023; Causil & Morais, 2023).

Table 1. Search results in the Scopus database (conducted in May 2023).

	Country	Uncertainty	Modelling	Indices	Comment
Kang et al. (2023)	Global	None/Partial	Life Cycle Analysis	Carbon price and CO_2 emissions	Life Cycle Analysis from statistical records
Huang et al. (2023)	China	None/Partial	Computable general equilibrium models	Energy efficiency improvement	Analysis from statistical records
Wei & Aaheim (2023)	Global	None/Partial	Computable general equilibrium models	Carbon tax and demand side management	Analysis from statistical records
Yeo & Oh (2023)	Global	None/Partial	Computable general equilibrium models	CO_2 emissions	Computable general equilibrium
Guang et al. (2023).	China	Partial	Demand side management of carbon emissions	Carbon price and CO_2 emissions	Analysis from statistical records
Sirin et al. (2023)	Global	None/Partial	Market failure or politics?	Carbon price and CO_2 emissions	Regulatory actions
Al Shammre et al. (2023)	OECD Countries	Partial	Analysis from statistical records	CO_2 emissions and taxes	Analysis from statistical records
Yang et al. (2023)	China	Fundamental	Machine learning	CO_2 emissions	Prediction from statistical records
Alizamir et al. (2023)	Iraq	Fundamental	Machine learning	Prediction by meteorological data	Machine learning
Hu & Cheng (2023)	China	Partial	Statistical Prediction	Carbon price and CO_2 emissions	Optimal prediction from statistical records
Wang et al. (20232a)	Global	None/Partial	Multi-objective strategy and optimisation	Carbon price and CO_2 emissions	Optimal prediction from statistical records
Wang et al. (2023b)	Global	None/Partial	Multi-objective strategy	Carbon price and CO_2 emissions	Data and Statistical Analyses
Liu & Ying (2023)	China	Fundamental	Multi-objective model	Carbon price and CO_2 emissions	Data Analysis
Rudnik et al. (2023)	EU	None/Partial	Supply side management with carbon price	Carbon price and CO_2 emissions	Prediction from statistical records

Source: The Authors.

In this context, Fuzzy Decision Making (FDM) theory, within the field of Artificial Intelligence, integrates human reasoning characteristics by subjectively evaluating and prioritising different criteria in decision making under fundamental uncertainty (Saaty, 2003; Camargo, 2019, 2021, 2022a, b, 2023; Liu et al., 2020). It extends classical decision theory to deal with uncertainty and imprecision in decision processes, using fuzzy logic to model fundamental uncertainty through fuzzy sets and membership (fuzzy preference) functions. This allows decisions to be made in situations where conditions and outcomes are not entirely clear or unambiguous. Fuzzy models incorporate human reasoning and perception, allowing the range of variables or functions under consideration to be adjusted to reflect the degree of acceptance of a variable in a given set. Exponential weights (EW) correspond to the preferences and hierarchical criteria of the decision maker. The solution should be the most satisfactory in terms of the decision maker's exponential weights and the accepted limits (upper and lower limits).

Optimisation methods are commonly classified according to their approach to solving, distinguishing between heuristic methods, which rely on rules of thumb or simplified search strategies, and algorithmic methods, which use precise algorithms to find optimal or suboptimal solutions. Another criterion is the type of search: uninformed search explores the search space without knowledge of the goal, while informed search uses goal-related information to guide the search towards more promising solutions. Optimisation methods can also be categorised by solution paradigm, distinguishing between knowledge-based methods that use predefined rules and relationships, data-driven methods that learn from data sets to make decisions, and optimisation-based methods that search for the best solution among a set of alternatives. At the intersection of AI and computational optimisation, Particle Swarm Optimisation (PSO), a metaheuristic inspired by collective dynamics in nature, has emerged as a powerful tool for finding optimal or near-optimal solutions to various optimisation problems. PSO involves both random and intelligent search by simulating social interactions between different possible solutions (particles). Each particle adjusts its position based on the best solution it has found and the best solution seen by the swarm, thus mimicking biological behaviour and using bio-inspired algorithms (Camargo et al., 2018; Camargo, 2019, 2021, 2022a, b, 2023).

In summary, the following challenges are highlighted and are still under discussion in the state-of-the-art (Camargo et al., 2018; Camargo, 2019, 2021, 2022b, 2023). The challenges of multidisciplinary or multi-objective optimisation when there is fundamental uncertainty; 2) The complexity of mathematical modelling due to the previous point, especially in terms of metric compatibility and their hierarchical evaluation. If one of the indices corresponds to an economic value and the other is a non-economic technical index, this implies an economic evaluation of this non-monetisable index, which may be unknown and not easy to obtain; 3) Due to the previous points, there is an inefficiency in determining the economic valuation (price of CO_2 and externality), especially when there are indices that cannot be monetised or are not subject to the laws of supply and demand on the markets; 4) Inadequate consideration of fundamental uncertainties in the current state of the art due to the difficulty or lack of consensus in the economic valuation of non-monetisable indices of the corresponding externality (positive or negative); 5) The best use of the aforementioned artificial intelligence tools to obtain and evaluate optimal solutions in this context.

Based on the above five state-of-the-art problems, this paper develops, analyses, generalises and validates a novel methodology to determine the optimal economic valuation of emissions (carbon price) within the Argentine production chain. It integrates fundamental uncertainty (Fuzzy Decision Making) and hierarchical variation (Analytic Hierarchy Process) with the optimisation of Supply Side Management (SSM) and Demand Side Management (DSM) through Life Cycle Assessment (LCA). Using Particle Swarm Optimisation (PSO), this approach provides a unique means of improving the efficiency (fuzzy intersection) of the Argentine production chain. This flexible methodology incorporates metaheuristics and hierarchisation for bi-objective (investment cost and emissions) optimisation under fundamental uncertainty, with the aim of optimising, evaluating, analysing and validating the results, taking into account the fuzzy preferences and priorities of the decision-makers (Offer Side and Demand Side Management Optimisation).

In this way, the methodology develops, generalises and analyses an index that belongs to the current research on the economic valuation of non-monetary attributes, known as the intrinsic cost index (Camargo et al., 2018; Camargo, 2023). In this work it is extended to cover different types of fuzzy intersection t-norms (with a coefficient p) and Exponential Weights (EW), and an additional term is added that allows modelling the resulting sign and thus determining the type of externality, taking into account positive and negative externalities. It provides an economic assessment of the emissions externality of the Offer Side Management (OSM) and Demand Side Management (DSM) optimisation. From this study, the efficiency frontiers of these cases are obtained (similar to the supply and demand model) and from there a new market equilibrium or computable general equilibrium model is presented as a result of this methodology. Then the theoretical, practical and economic contributions the present methodology of this paper are elaborated, analysed, generalised and validated in the following sections, together with an assessment to ensure coherence, realism and consistency with existing economic theories of market equilibrium.

This work is structured as follows. Section 2. summarises the material and methods: The Fuzzy Decision Making Theory and Analytic Hierarchy Process (Section 2.1.), Particle Swarm Optimisation (Section 2.2.), Life Cycle Analysis with Demand Supply and Demand Side Management (Section 2.3.) and Computable General Equilibrium Model (Section 2.4.). Section 3 develops the optimal economic evaluation of the Argentinean production chain with contributions and results: proposed novel methodology to obtain the optimal economic value of emissions with uncertainty (Section 3.1.), Theoretical and practical contributions of the proposed methodology (Section 3.2.), Practical contributions of the proposed methodology with the comparison of the two cases related to Supply and Demand Side Management (Section 3.3.) and Economic Contributions of the proposed methodology (Section 3.4.). Section 3.4. develops the Generic Camargo Intrinsic Cost (see Section 3.4.1. and Section 3.4.2.). Finally, Section 4 presents the conclusions of this work.

2. Materials and methods

2.1. Fuzzy decision making theory and analytic hierarchy process

Fuzzy decision theory (Figure 1 and Equation 1) is based on human behaviour to make decisions (decision maker) based on a criterion of exponential weights and evaluation under uncertainty (fuzzy function). An acceptance (fuzzy) function is used to transform a set of indices to the fuzzy domain (Saaty, 2003; Liu et al., 2020; Camargo 2023). In Equation 1, the EW_m are Exponential Weights (that are obtained for each index or attribute m) whose effect is to expand ($EW_m < 1$) and contract ($EW_m > 1$) the fuzzy preference function. If it is desired to increase the index, the function has a positive slope (and vice versa). The index of the preference function μ_m (Equation 1) is associated with the degree of acceptance of the evaluated attribute or index by the decision maker (economic costs and CO_2 emissions), according to his or her established exponential weights or EW. For each objective (or constraint) function calculated (emissions and investment cost), the fuzzy functions (Equation 1) associated with its objective or constraint are defined as follows: consider an upper and a lower bound in the possible values of the variable corresponding to a given objective or constraint m, U_m. These

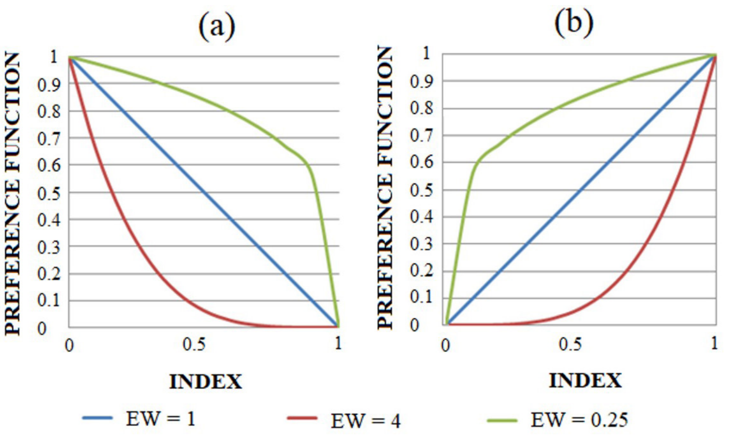

Figure 1. Preference function: (a) Decrease of U_m and (b) Growth of U_m.
Source: The authors.

indices are related by the fuzzy intersection (product) operation of the preference functions μ_m and since these are dimensionless, there is no metric compatibility problem.

$$\mu_m = \begin{cases} 1 \cup 0 & , U_m^{Low} \ge U_m \\ \left(\frac{U_m^{Up} - U_m}{U_m^{Up} - U_m^{Low}}\right)^{EW_m} \cup \left(\frac{U_m - U_m^{Low}}{U_m^{Up} - U_m^{Low}}\right)^{EW_m} & , U_m^{Low} \le U_m \le U_m^{Up} \; . \\ 0 \cup 1 & , U_m^{Up} \le U_m \end{cases} \qquad (1)$$

Within the theory of fuzzy decision making, the fuzzy product type intersection is used to model human reasoning in situations where relationships between variables are not precise or subject to uncertainty, and it allows to represent how different variables influence each other in a gradual and not necessarily binary way. From a human reasoning perspective, the fuzzy product type intersection reflects the ability of society (supply and demand management) to consider multiple factors and evaluate how they interact to make decisions. The fuzzy intersection or t-norm product tp (see Equation 2, Equation 5 and Equation 8) is the most common confluence (fuzzy operator) and this makes it possible to model how individual exponential weights, objectives, constraints and other factors combine in an incremental way to influence the final outcome (investment cost and emissions). This more accurately reflects how people weigh (exponential weights) and balance (fuzzy intersection) different considerations when making real-life decisions (Fuzzy Decision Making). Fuzzy intersections include the Einstein product, the algebraic product and the family of t-norms Hamacher's product, which depend on a factor p. These t-norms are further developed in Section 3.2.1.

2.2. Particle Swarm Optimisation (PSO)

Particle Swarm Optimisation (PSO) is a bio-inspired optimisation algorithm based on the social and foraging behaviour of swarms of animals, such as birds or fish. From an artificial intelligence point of view, PSO is inspired by observing how members of a swarm cooperate and communicate with each other to find the best possible solution to a problem. In PSO, each "particle" represents a possible solution to the optimisation problem, and the swarm of particles moves through the search space to find the optimal solution (in this work, it is the t-norm). In this work, as presented in Section 3, the particle corresponds to the optimal allocation of resources in demand and supply management according to the life cycle analysis performed. Each particle adjusts its position based on its own experience (personal best position) and the collective experience of the swarm (global best position). This process is repeated iteratively until a predefined stopping condition or iteration limit is reached. From a performance point of view, PSO is a metaheuristic algorithm that is not guaranteed to find the globally optimal solution, but is highly efficient at finding near-optimal solutions to problems with high dimensionality or multiple local optima.

As the particles move through the search space (the set of possible solutions to the problem), the PSO uses the information exchanged between them to guide the search towards promising regions (and best solutions). In summary, from an AI perspective, PSO is a nature-inspired optimisation technique that exploits swarm behaviour to find optimal solutions in complex search spaces. Its operation is based on cooperation and communication between swarm particles, making it an effective approach to solving a wide range of optimisation problems. Due to space limitations, the Particle Swarm Optimisation metaheuristic is not developed in depth in this paper, but can be found in the following references (Casanova et al., 2018; Camargo et al., 2019; Camargo, 2019, 2021, 2022a, b, 2023).

2.3. Life Cycle Analysis, Demand Supply Management and Offer Supply Management

Life Cycle Assessment (LCA) is a tool used to evaluate the environmental impact of a product or service throughout its life cycle, from the extraction of raw materials to its final disposal. It examines all environmental aspects, such as the use of natural resources, air emissions, waste generation and energy consumption. In the context of Demand Supply Management and Offer Supply Management, LCA provides critical information for supply chain decision making. Demand Supply Management (DSM): involves managing the demand for products or services based on market needs and resource availability. LCA can help companies understand how products affect the environment throughout their life cycle, which can influence consumer choice and therefore demand for more sustainable products. Offer Supply Management (OSM): This refers to the management of the supply of products or services, including supplier selection and supply chain optimisation. LCA can be used to assess the environmental impacts of materials and processes used by suppliers, helping companies to make informed decisions about supplier selection and supply chain management to improve sustainability (Camargo & Schweickardt, 2014; Kang et al., 2023; Huang et al., 2023; Wei & Aaheim, 2023; Yeo & Oh, 2023; Guang et al., 2023; Sirin et al., 2023; Al Shammre et al., 2023; Yang et al., 2023; Alizamir et al., 2023; Hu & Cheng, 2023; Wang et al., 2023a; Liu & Ying, 2023; Rudnik et al., 2023).

2.4. Computable General Equilibrium Model

The Computable General Equilibrium Model plays a crucial role in understanding the dynamics of supply and demand within the production chain, particularly in the context of economic uncertainty. This model serves as a computational framework that simulates the interactions between various sectors of the economy, capturing how changes in one sector affect others through intricate feedback loops. However, determining the equilibrium of such a model in the face of uncertainty poses significant challenges, as it requires predicting the behavior of numerous interconnected factors amidst fluctuating conditions. In essence, while the model provides valuable insights into the overall equilibrium of the production chain, navigating uncertainty adds layers of complexity to its determination, emphasizing the need for robust analytical tools and methodologies to address such challenges effectively (Kang et al., 2023; Huang et al., 2023; Wei & Aaheim, 2023; Yeo & Oh, 2023; Guang et al., 2023; Sirin et al., 2023; Al Shammre et al., 2023; Yang et al., 2023; Alizamir et al., 2023; Hu & Cheng, 2023; Wang et al., 2023a; Liu & Ying, 2023; Rudnik et al., 2023).

3. Optimal economic evaluation of Argentinean production chain with contributions and results

This section presents the main developments of the theoretical, practical and economic results and contributions of the proposed methodology. Firstly, the proposed novel methodology for obtaining the optimal economic value of emissions with uncertainty is developed (Section 3.1). In this context, it is shown that the fuzzy intersection function of the Hamacher family contains three types of t-norms, which allows a generalisation and simplification of the intrinsic cost index. It is reiterated that this index allows the economic valuation of variables that are not directly monetisable and have no associated market. In this way, some modifications have been made to the formulation of the index in order to reduce mathematical ambiguities and to clarify its conceptual and practical application in obtaining externalities.

Secondly, the practical contributions of the present methodology to the two cases of environmental impact reduction discussed in this paper (Section 3.2) are elaborated. These two cases are Supply Side Management (SSM) and Demand Side Management (DSM). These two cases are optimised by Particle Swarm Optimisation using a hierarchical approach facilitated by the Analytic Hierarchy Process (a variation of exponential weights associated with hierarchy). It will be shown that this model, a novel outcome of this proposal, incorporates economic valuation of non-monetisable attributes (intrinsic cost index developed in Section 3.1) and addresses fundamental uncertainty (fuzzy decision making developed in Section 3.1). Furthermore, as developed in section 2.2, it includes both objective evaluation (maximum and minimum set of quality criteria) and subjective evaluation (types of t-standards and satisfaction levels or preference functions). Optimal curves are presented for each exponential weighting value (hierarchy), allowing the construction of efficiency frontiers for both cases (Section 3.2). This analysis contributes to the following section.

Thirdly, the theoretical, practical and economic contributions of the present methodology in the Argentinean production chain are presented through the presentation of a new market equilibrium model (Section 3.3). It will be shown that this equilibrium model allows to obtain the break-even point of the estuary prospects, according to the model of supply and demand curves. It is also shown that the efficiency frontier of intrinsic costs is associated with marginal costs. In this way, an introduction is developed that paves the way for future models that allow the objective and subjective valuation of both positive and negative externalities, subject to fundamental uncertainty, in line with the efficiency frontier. The important practical implications are highlighted.

3.1. Proposed novel methodology to obtain the optimal economic value of emissions with uncertainty

Firstly, a Life Cycle Assessment (LCA) is carried out, considering the material and fuel flow of the Argentine Production Chain (APC) through the following stages: resource extraction, material processing, manufacturing, construction, transportation and waste management (Figure 2). In this way, the parameters (including technical data) were mainly processed and the complete model was validated using information from public reports available in the database of the Ministry of Energy and Mines (Camargo & Schweickardt, 2014; Camargo, 2019, 2021, 2022a, b, 2023; Argentina, 2023). Two cases are considered in this optimisation: Supply Side Management (SSM) and Demand Side Management (DSM). Supply Side Management (SSM) efficiency optimisation involves reducing emissions by investing in energy efficient improvements in the production chain or by changing consumption patterns with minimal investment. Demand Side Management (DSM) efficiency improvement involves reducing investment and emissions by implementing measures that encourage conservation.

Secondly, based on the production chain model (Life Cycle Analysis model - LCA), the efficiency index to be optimised (fuzzy intersection) is determined and the attributes to be evaluated (investment costs and CO_2 emissions) are defined. The decision maker (Figure 2) then evaluates the indices (emissions and investment costs) resulting from the Life Cycle Analysis and transforms them into the fuzzy domain, taking into account the upper and lower bounds (see Equation 1) obtained from the upper and lower limits of the analysed Life Cycle Analysis (U_m^{Up} and U_m^{Low}).

Thirdly, to find the optimal solution, this methodology uses fuzzy decision theory and analytic hierarchy process with artificial intelligence tools such as particle swarm optimisation metaheuristics. It provides optimal solutions for supply chain management and industrial processes. The values of the optimal attributes (investment cost and emissions) depend on the solution proposed by the Particle Swarm Optimisation (PSO) metaheuristic and the prioritisation proposed by the Analytic Hierarchy Process. In addition, the hierarchy (Analytic Hierarchy Process) and the upper and lower bounds (static) of these attributes are determined. From there, the evaluation of the indices is carried out, which also depends on the direction of improvement of the function (increase or decrease). Figure 2 analyses two cases for reducing environmental impacts (emissions), where these cases are made according to the decision maker's cases (Supply Side Management and Demand Side Management) on emissions and the cost of the investment (carbon price).

The PSO is used to achieve consistent levels of efficiency (constant t-norm) in the optimisation of the Argentinean production chain and allows the generation of customised, flexible and optimal solutions with objectives and constraints determined by the prioritisation of the Analytic Hierarchy Process. In this sense, this metaheuristic aims to maximise both the fuzzy indices and their intersection, regardless of whether they are at maximum or minimum values. This flexibility allows the use of heuristics that are not constrained by this aspect. In other words, if the aim is to maximise the given indices then the value of the fuzzy upper limit (U_m^{Up}) will be sought to be reached. Conversely, if the aim is to minimise the indices then the value of the fuzzy lower limit (U_m^{Low}) will be sought to be reached (see Equation 1).

Fourthly, the economic valuation (Intrinsic Cost) of CO_2 emissions (carbon tax) from the Argentinean production chain has been compared with external taxes (Kyoto Protocol), taking into account the both cases (demand side management and supply side management), calculated using the PSO. This economic evaluation integrates both objective (maximum U_m^{Up} and minimum limits U_m^{Low}) and subjective (acceptance μ_m, hierarchy EW_m and uncertainty) assessments through the Generic Camargo Intrinsic Cost Index and its methodology.

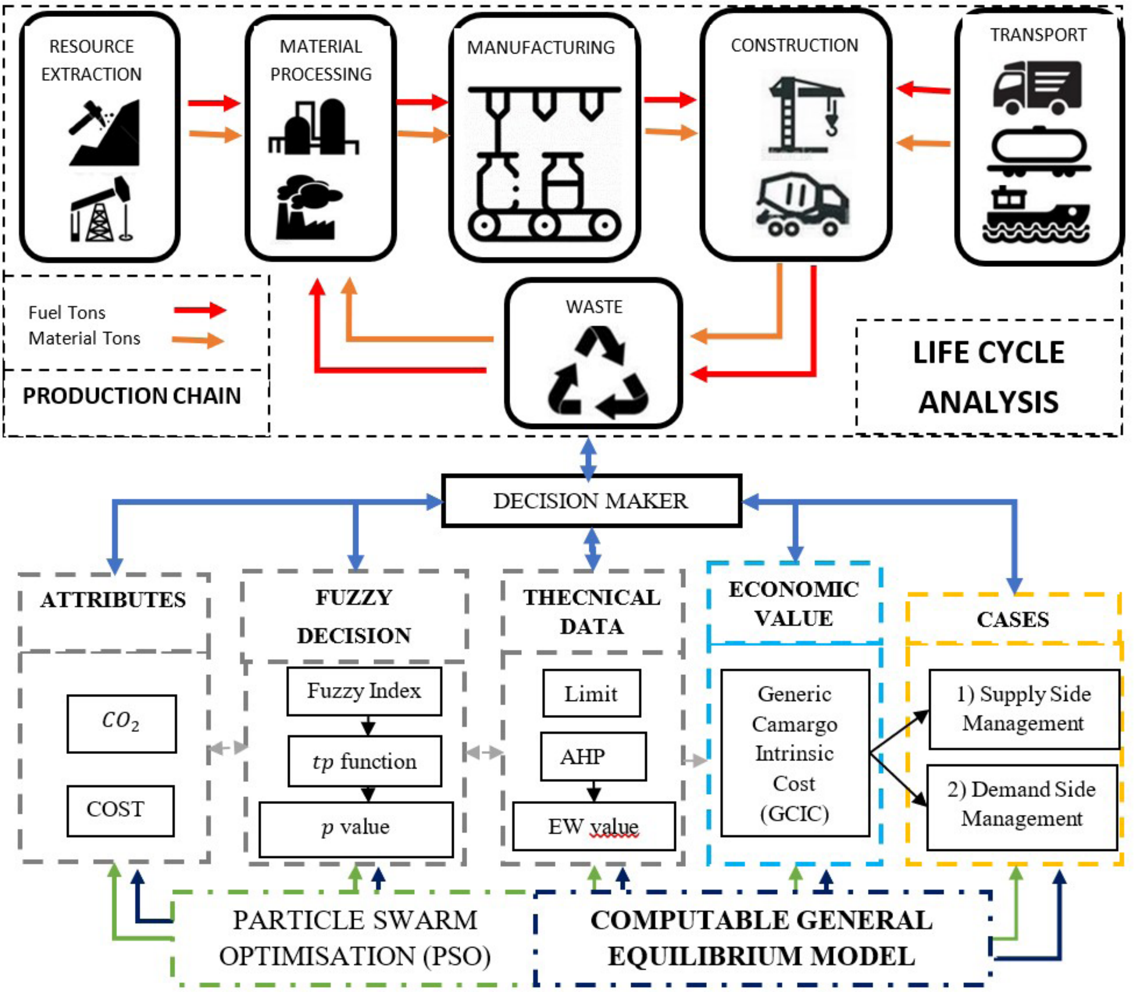

Figure 2. Proposed novel methodology: Optimal Economic Value (Generic Camargo Intrinsic Cost - GCIC) of Supply and Demand Side Management in terms of Life Cycle Analysis of the Argentinean Production Chain under Fundamental Uncertainty (Fuzzy Decision Making) and Hierarchy Variation (Analytic Hierarchy Process - AHP) with the Exponential Weights (EW) by means of Particle Swarm Optimisation.
Source: The authors.

The calibration and validation of this index was therefore based on energy and CO_2 emission records from the Argentine government and market data. The Exponential Weights (EW) represent the hierarchy of decision makers (Supply and Demand Side Management) for the attributes being assessed and influence the economic valuation (carbon price) based on the assigned hierarchy (Section 3.3).

Fifthly, from the four aspects mentioned above, the curves associated with the attributes analysed and the Generic Camargo Intrinsic Cost and the efficiency frontiers separating the feasible and non-feasible areas are obtained. This results in a new computable general equilibrium model in which the equilibrium points and feasible zones are searched and the marginal cost of equilibrium is obtained, as presented in Section 3.4.

3.2. Theoretical, practical and economic contributions of the proposed methodology

3.2.1. Theoretical contributions in Fuzzy decision making theory: t-norms product

In this section it is shown from the t-norm that both the Einstein product and the algebraic product are special cases of the Hamacher product family. This is an advantage as it simplifies the procedures used when working with one or the other t-norm. The procedure for obtaining the final efficiency index and the associated intrinsic cost is simplified by using a general equation that covers the different particular cases for the intersection operators. It is recalled that the functions used are continuous and derivable, which makes it possible to obtain the costs associated with their variation.

As mentioned in Section 2.1, $tp\left(\mu_i,\mu_j\right)$ is an operator (generally referred to as the t-norm) between the values of the membership functions (μ_i and μ_j). In the state of the art there are several t-norms (Equation 2, Equation 5 and Equation 8): the algebraic product, the Einstein product and the particular Hamacher product. These t-norms have the interesting property of being differentiable, which the t-min does not have. This property allows us to obtain the impact of an objective that is analysed in the other cases, namely the social cost. The constant p (Equation 2) is a parameter that, depending on its value, defines a family of curves. As a result, the final intersection will be more demanding or laxer, depending on its value (Camargo et al., 2018; Camargo, 2019, 2021, 2022a, b, 2023).

EINSTEIN PRODUCT: The Einstein Product (tp_E) is a special case of the Hamacher family product (tp_H), as shown below (Equation 2, Equation 3 and Equation 4).

$$tp_E = tp_H \Rightarrow \frac{\mu_i \cdot \mu_j}{2-\left(\mu_i+\mu_j-\mu_i \cdot \mu_j\right)} = \frac{\mu_i \cdot \mu_j}{p+(1-p)\cdot\left(\mu_i+\mu_j-\mu_i \cdot \mu_j\right)} \tag{2}$$

$$2-\left(\mu_i+\mu_j-\mu_i \cdot \mu_j\right) = p+(1-p)\cdot\left(\mu_i+\mu_j-\mu_i \cdot \mu_j\right) \tag{3}$$

$$p\cdot\left(1-\left(\mu_i+\mu_j-\mu_i \cdot \mu_j\right)\right) = 2\cdot\left(1-\left(\mu_i+\mu_j-\mu_i \cdot \mu_j\right)\right) \Rightarrow p=2 \tag{4}$$

ALGEBRAIC PRODUCT: The Algebraic Product (tp_A) is a special case of the Hamacher family product (tp_H), as shown below (Equation 5, Equation 6 and Equation 7).

$$tp_A = tp_H \Rightarrow \mu_i \cdot \mu_j = \frac{\mu_i \cdot \mu_j}{p+(1-p)\cdot\left(\mu_i+\mu_j-\mu_i \cdot \mu_j\right)} \tag{5}$$

$$1 = p+(1-p)\cdot\left(\mu_i+\mu_j-\mu_i \cdot \mu_j\right) \tag{6}$$

$$p\cdot\left(1-\left(\mu_i+\mu_j-\mu_i \cdot \mu_j\right)\right) = 1-\left(\mu_i+\mu_j-\mu_i \cdot \mu_j\right) \Rightarrow p=1 \tag{7}$$

PARTICULAR HAMACHER PRODUCT: The Particular Hamacher Product (tp_{H0}) is a special case of the Hamacher family product (tp_H), as shown below (Equation 8, Equation 9 and Equation 10).

$$tp_{H0} = tp_H \Rightarrow \frac{\mu_i \cdot \mu_j}{\mu_i+\mu_j-\mu_i \cdot \mu_j} = \frac{\mu_i \cdot \mu_j}{p+(1-p)\cdot\left(\mu_i+\mu_j-\mu_i \cdot \mu_j\right)} \tag{8}$$

$$\mu_i+\mu_j-\mu_i \cdot \mu_j = p+(1-p)\cdot\left(\mu_i+\mu_j-\mu_i \cdot \mu_j\right) \tag{9}$$

$$p\cdot\left(1-\left(\mu_i+\mu_j-\mu_i \cdot \mu_j\right)\right) = 0 \Rightarrow p=0 \tag{10}$$

Through this analysis, a general and simplified expression of the intrinsic cost index is obtained and developed in the following section. Thus, in this article it is shown that the t-norms Einstein product ($p=2$), Particular Hamacher Product ($p=1$) and Algebraic product ($p=0$) are special cases of the Hamacher product family. This discovery allows this methodology to define a general intrinsic cost index for any fuzzy t-norm of the product type.

3.2.2. Intrinsic Cost Index based on Fuzzy Decision Making Theory

Intrinsic Cost (IC) index: The economic evaluation of each criterion is given by the intrinsic cost index (Equation 11). The Intrinsic Cost Index (IC) takes into account the economic valuation of non-monetary indices based on an objective valuation (incremental valuation based on Upper and Lower limits), subjective and hierarchical (Analytic Hierarchy Process). The intrinsic cost index corresponds to the derivative of one index with respect to the other (cost of emissions with respect to emissions), which is not easy to determine due to the presence of uncertainty and subjectivity. In the past, this index was only obtained for the t-norm of Einstein's product and with another index for electricity systems developed by Schweickardt & Pistonesi (2010) and improved by Camargo et al. (2018, 2023a, b); Camargo (2019, 2021, 2022a, b, 2023). In Equation 11, the intrinsic cost (IC) then represents the slope of the efficiency frontier of the functional relationship between the specified attributes, and the sign ± depends on whether it's a growth (+) or a decrease (-).

$$IC_{j\,i} = \frac{\partial U_j}{\partial U_i} = \frac{\partial U_j}{\partial \mu_j} \cdot \frac{\partial \mu_j}{\partial \mu_i} \cdot \frac{\partial \mu_i}{\partial U_i} \qquad \forall \qquad \frac{\partial \mu_{i/j}}{\partial U_{i/j}} = \pm \frac{EW_{i/j}}{U_{i/j}^{Up} - U_{i/j}^{Low}} \cdot \mu_i^{\left(1 - \frac{1}{EW_{i/j}}\right)} \tag{11}$$

With this method, the variable U_j, which is related to criterion j (investment costs), is the mathematical derivative of the variable U_i, which is generally related to index i (emissions). In this way, the term $\frac{\partial \mu_j}{\partial \mu_j}$ corresponds to the functional derivative of the fuzzy function $\partial \mu_j$ respect to the fuzzy function $\partial \mu_i$ (see Equation 1). If both attributes (i and j) correspond to decrease (see Figure 1a) or growth (see Figure 1b) then it will have a (+) sign, in any other case it will be (-). The latter influences the sign of the index and determines whether it is a penalty or an incentive (see Section 3.3.1 and Section 3.3.2.).

Schweickardt Intrinsic Cost (SIC) proposal: The first proposal (Schweickardt & Pistonesi, 2010) have obtained this term considering the t-norm Einstein Product of all attributes. If there are 4 attributes (Equation 12, Equation 13 and Equation 14) and the t-norm Einstein Product, then the t-norms of the attributes are obtained, applying the properties of the t-norms.

$$t(\mu_1, \mu_2) = \frac{\mu_1 \cdot \mu_2}{2 - (\mu_1 + \mu_2 - \mu_1 \cdot \mu_2)} \tag{12}$$

$$t(\mu_3, t(\mu_1, \mu_2)) = \frac{\mu_3 \cdot t(\mu_1, \mu_2)}{2 - (\mu_3 + t(\mu_1, \mu_2) - \mu_3 \cdot t(\mu_1, \mu_2))} \tag{13}$$

$$t(\mu_4, t(\mu_3, t(\mu_1, \mu_2))) = \frac{\mu_4 \cdot t(\mu_3, t(\mu_3, t(\mu_1, \mu_2)))}{2 - (\mu_4 + t(\mu_3, t(\mu_3, t(\mu_1, \mu_2))) - \mu_4 \cdot t(\mu_3, t(\mu_3, t(\mu_1, \mu_2))))} \tag{14}$$

From this expression, it is necessary to solve for $\mu_j = f(\mu_i)$, for any two attributes i and j, and then obtain the functional derivative $\frac{\partial \mu_j}{\partial \mu_i}$ (holding other attributes constant). The result is a long and complex mathematical equation with only 4 attributes to analyse (Equation 15 to Equation 17), which is impractical for problems with more objectives and constraints. To show the complexity of this proposal, the following expression is defined for n fuzzy functions and auxiliary indices r, s, q and z:

$$tp(\mu \ldots \mu_r \ldots \mu_q \ldots \mu_z \ldots \mu_n) = \frac{\prod_{q=}^{n} \mu_q}{\prod_{r=2}^{4}(2 - \mu_r) - \sum_{s=1}^{2}\left(\prod_{z=s+2}^{n}(2 - \mu_z)\right) \cdot \left(\prod_{z=1}^{s} \mu_z - \prod_{z=1}^{s+1} \mu_z\right)} \tag{15}$$

$$\mu_j = f(\mu_1 \ldots \mu_r \ldots \mu_q \ldots \mu_z \ldots \mu_n, tp(\mu_1 \ldots \mu_r \ldots \mu_q \ldots \mu_z \ldots \mu_n)) \tag{16}$$

$$\frac{\partial \mu_j}{\partial \mu_i} = \frac{\partial}{\partial \mu_i} f(\mu_1 \ldots \mu_i \ldots \mu_n, tp(\mu_1 \ldots \mu_r \ldots \mu_q \ldots \mu_z \ldots \mu_n)) \quad \forall \quad \frac{\partial}{\partial \mu_i} tp(\mu_1 \ldots \mu_n) = 0 \tag{17}$$

It is observed in Equation 15 to Equation 17 that the function is complex to solve for multiple fuzzy functions (Schweickardt & Pistonesi, 2010), therefore this proposal is not feasible. Therefore, the Schweickardt intrinsic cost proposal is written in Equation 18. The equation has been simplified by including one function due to its infeasibility for more than 2 attributes.

$$SIC_{ij} = \pm\left(\frac{EW_i}{EW_j}\right)\cdot\left(\frac{U_j^{Up}-U_j^{low}}{U_i^{Up}-U_i^{low}}\right)\cdot\left(\frac{\mu_i^{\left(1-\frac{1}{EW_i}\right)}}{\mu_j^{\left(1-\frac{1}{EW_j}\right)}}\right)\cdot\frac{\partial}{\partial\mu_i}f\left(\mu_1\ldots\mu_i\ldots\mu_n,tp\left(\mu_1\ldots\mu_n\right)\right) \quad (18)$$

Camargo Intrinsic Cost (CIC) proposal: An improved proposal of the current research line called Camargo Intrinsic Cost (CIC) is presented here. In Equation 19, since the Ceteris Paribus clause is applied, then it is only necessary to compare two attributes, since all the others will be constant. Then, the next step is to derive the term $\mu(\mu_i,\mu_r)$ respect to μ_i, it is applyiing the clause 'Ceteris Paribus' (Camargo et al., 2018; Camargo, 2023).

$$tp = \frac{\mu_i\cdot\mu_j}{2-\left(\mu_i+\mu_j-\mu_i\cdot\mu_j\right)} \Rightarrow \frac{\partial\mu_j}{\partial\mu_i} = -\frac{\mu_j}{\mu_i}\cdot\frac{\mu_j-2}{\mu_i-2}\ \forall\ \frac{\partial \mathrm{tp}}{\partial\mu_i}=0 \quad (19)$$

The Camargo Intrinsic Cost is given by Equation 20 (Einstein product):

$$CIC_{ij} = \pm\left(\frac{EW_i}{EW_j}\right)\cdot\left(\frac{U_j^{Up}-U_j^{low}}{U_i^{Up}-U_i^{low}}\right)\cdot\left(\frac{\mu_i^{\left(1-\frac{1}{EW_i}\right)}}{\mu_j^{\left(1-\frac{1}{EW_j}\right)}}\right)\cdot\left(\frac{\mu_j}{\mu_i}\right)\cdot\left(\frac{\mu_j-2}{\mu_i-2}\right)\forall\,\mathrm{tp} = \frac{\mu_i\cdot\mu_j}{2-\left(\mu_i+\mu_j-\mu_i\cdot\mu_j\right)} \quad (20)$$

Generic Camargo Intrinsic Cost (GCIC) proposal: This paper presents a novel, improved and generic proposal called Camargo Intrinsic Cost (CIC). To simplify the algorithm, an auxiliary variable β_m is added to the definition of the fuzzy functions (Equation 21 and Equation 22), depending on whether it is a decrease (Figure 2a) or a growth (Figure 2b).

BEGIN /* Fuzzy decision making with this new methodology */

Data: Objective and Constraint indices U_m, Exponential Weights EW_m (AHP), Lower U_m^{Low} and Upper U_m^{Up} Limits.

FOR $(\mathrm{m}=1:2)$ DO

Step 1: Calculate the auxiliary variable β according to the Equation 21.

$$\beta = \begin{cases} 1 & U_m \quad growth \\ 0 & decrease\, of\, U_m \end{cases} \quad (21)$$

Step 2: Calculate the states μ_m using the next function according to the Equation 22.

$$\mu_m = \begin{cases} 1-\beta_m & ,U_m^{Low} \geq U_m \\ \left(\left(\frac{U_m^{Up}-U_m}{U_m^{Up}-U_m^{Low}}\right)\cdot\left(1-\beta_m\right)+\left(\frac{U_m-U_m^{Low}}{U_m^{Up}-U_m^{Low}}\right)\cdot\beta_m\right)^{EW_m} & ,U_m^{Low}\leq U_m \leq U_m^{Up} \\ \beta_m & ,U_m^{Up}\leq U_m \end{cases} \quad (22)$$

END FOR

Step 3: Calculate $\mathrm{tp}\left(\mu_i,\mu_j\right)$ using the chosen t-norm, where $i=1$ is the CO_2 emissions and $j=2$ is the Investment Cost according to the Equation 23.

$$tp\left(\mu_i,\mu_j\right)=\frac{\mu_i\mu_j}{p+(1-p)\cdot\left(\mu_i+\mu_j-\mu_i\cdot\mu_j\right)}=\begin{cases}\dfrac{\mu_i\cdot\mu_j}{2-\left(\mu_i+\mu_j-\mu_i\cdot\mu_j\right)} & ,p=2\\ \vdots\vdots & ,p=1\\ \mu_i\cdot\mu_j & ,p=0\\ \dfrac{\mu_i\cdot\mu_j}{\mu_i+\mu_j-\mu_i\cdot\mu_j} & \vdots\\ \dfrac{\mu_i\cdot\mu_j}{p+(1-p)\cdot\left(\mu_i+\mu_j-\mu_i\cdot\mu_j\right)} & \text{, Another Case}\end{cases} \tag{23}$$

END PROGRAM

Mathematically deriving Equation 22 (where m equal to i or j) with respect to $U_{i/j}$ gives Equation 24.

$$\frac{\partial\mu_{i/j}}{\partial U_{i/j}}=\left(2\cdot\beta_i-1\right)\cdot\frac{EW_{i/j}}{U_{i/j}^{Up}-U_{i/j}^{Low}}\cdot\mu_i^{\left(1-\frac{1}{EW_{i/j}}\right)} \tag{24}$$

From the t-norm of the Generic Hamacher product, the mathematical derivative of the μ_j membership function with respect to μ_i is obtained in Equation 25, where $i=1$ is the CO_2 emissions and $j=2$ is the Investment Cost.

$$\frac{\partial\mu_j}{\partial\mu_i}=-\left(\frac{p+(1-p)\cdot\mu_j}{p+(1-p)\cdot\mu_i}\right)\cdot\left(\frac{\mu_j}{\mu_i}\right)\ \forall\left(\frac{\partial \text{tp}}{\partial\mu_i}=\frac{\partial}{\partial\mu_i}\left(\frac{\mu_i\cdot\mu_j}{p+(1-p)\cdot\left(\mu_i+\mu_j-\mu_i\cdot\mu_j\right)}\right)=0\right) \tag{25}$$

According to Equation 25, Equation 26 is obtained, where: 1) if it is true that $p=0$ then the intrinsic cost of the particular Hamacher product is obtained, 2) if it is true that $p=1$ then the intrinsic cost of the algebraic product is obtained and 3) if it is true that $p=2$ then the intrinsic cost of the Einstein product is obtained. This is logical because replacing these values of p in Equation 26 gives the corresponding t-norms. The negative sign indicates that, with constant efficiency, if one fuzzy index increases, the other must decrease and vice versa, which is true for all types of t-norms and fuzzy indices. Equation 27 then shows the new Generic Camargo Intrinsic Cost for Fuzzy Linear Function (see Equation 1) and the Hamacher family.

$$\frac{\partial\mu_j}{\partial\mu_i}=\begin{cases}-\left(\dfrac{\mu_j}{\mu_i}\right)^2,|\ p=0\\ -\left(\dfrac{\mu_j}{\mu_i}\right),|\ p=1\\ -\left(\dfrac{\mu_j-2}{\mu_i-2}\right)\cdot\left(\dfrac{\mu_j}{\mu_i}\right),|\ p=2\\ -\left(\dfrac{p+(1-p)\cdot\mu_j}{p+(1-p)\cdot\mu_i}\right)\cdot\left(\dfrac{\mu_j}{\mu_i}\right),|\text{another case}\end{cases} \tag{26}$$

$$GCIC_{ij}=\underbrace{\left(-\frac{2\cdot\beta_i-1}{2\cdot\beta_j-1}\right)}_{sign}\underbrace{\left(\frac{EW_i}{EW_j}\right)}_{a_{ij}}\underbrace{\left(\frac{U_j^{Up}-U_j^{low}}{U_i^{Up}-U_i^{low}}\right)}_{b_{ij}}\underbrace{\left(\frac{\mu_i^{\left(1-\frac{1}{EW_i}\right)}}{\mu_j^{\left(1-\frac{1}{EW_j}\right)}}\right)}_{c_{ij}}\underbrace{\left(\frac{p+(1-p)\cdot\mu_j}{p+(1-p)\cdot\mu_i}\right)\left(\frac{\mu_j}{\mu_i}\right)}_{d_{ij}}=\pm\left|GCIC_{ij}\right| \tag{27}$$

The resulting sign indicates the type of externality it has, it is a positive externality (Case 1) if it satisfies $\beta_i \neq \beta_j$ and it is a negative externality (Case 2) if it satisfies $\beta_i = \beta_j$. The term a_{ij} is the preference ratio (AHP), which gives the relative evaluation between the evaluated indices (CO_2 emissions and investment costs). The EWs are obtained from an analysis of the Perron eigenvalue of the preference matrix, which is equivalent to the AHP (Camargo, 2023). The term b_{ij} is the incremental cost of an attribute j with respect to another of interest i (CO_2). The term c_{ij} is related to the influence of the acceptance of attributes and the hierarchy of decision makers. It should be noted that if the exponential weights were one, i.e. if there were no contraction or dilation of the preference functions due to their non-hierarchisation, then this term would be one and would disappear from the equation. In other words, if there is no over- or undervaluation of the fuzzy attributes, there will be no over- or under-cost (in our economic case). The term d_{ij} corresponds to the mathematical derivative of the fuzzy function $\partial\mu_j$ with respect to the fuzzy function $\partial\mu_j$, which is associated with the slope of the functional relationship of the preference functions. The factor b_{ij} determines the objective valuation (incremental cost) of attribute j, while a_{ij}, c_{ij} and d_{ij} determine the subjective valuation.

The equation is improved by including a factor that determines the sign as a function of the parameters used to model the sign of the slope of the fuzzy functions (β_i and β_j). In summary, this improvement of the index takes into account the three types of t-norm studied in this line of research, in addition to clearly incorporating their sign, which simplifies their analysis and practical application in a comparative study.

Generic Camargo Intrinsic Cost (GCIC) analysis: An improved and novel proposal of the current research line called Generic Camargo Intrinsic Cost (GCIC) is presented here. The analysis of the generic Camargo intrinsic cost expression is summarised in Equation 28, where it is observed that if the absolute and relative valuation terms are very high, then the economic valuation of attribute i will be high with respect to j (see Section 3.4.1. and Section 3.4.2.).

If a_{ij}, b_{ij}, c_{ij} and d_{ij} are equal to zero, then the economic valuation of attribute j will be zero ($\left| GCIC_{ij} \right| = 1$). If a_{ij}, b_{ij}, c_{ij} and d_{ij} are equal to the one unit, then the fuzzy function for the two attributes (μ_i and μ_j) will have the same value ($\mu_i = \mu_j$), so the economic valuation will be one ($\left| GCIC_{ij} \right| = 1$). If a_{ij}, c_{ij} and d_{ij} are equal to the one unit, then a constant value b_{ij} will then be obtained that is independent of the evaluated indices, i.e. the intrinsic cost will be an incremental cost between the extreme values ($\left| GCIC_{ij} \right| = b_{ij} = \frac{U_j^{Up} - U_j^{low}}{U_i^{Up} - U_i^{low}}$). If additionally it is satisfied that ($U_j^{low} = U_i^{low} = 0$) the intrinsic cost will be equal to the average cost (or marginal cost) of the maximum values ($\left| GCIC_{ij} \right| = \frac{U_j^{Up}}{U_i^{Up}}$).

Therefore, based on this theoretical analysis, the proposed index can be used as an economic indicator to value non-monetary attributes and as a regulatory mechanism. In the context of the Coase theorem and property rights, it is the price at which the bonds must be traded according to the negative externality (Case 2) produced and, it is a tax to be paid by the production chain. In the case of a positive externality (Case 1), it is the subsidy received as an incentive or compensation for the emissions benefit produced.

$$\left| GCIC_{ij} \right| = \begin{cases} \infty, \left(a_{ij} \gg 1\right) \cup \left(b_{ij} \gg 1\right) \cup \left(c_{ij} \gg 1\right) \cup \left(d_{ij} \gg 1\right) \\ 0, \left(a_{ij} = 0\right) \cup \left(b_{ij} = 0\right) \cup \left(c_{ij} = 0\right) \cup \left(d_{ij} = 0\right) \\ 1, \left(a_{ij} = 1\right) \cap \left(b_{ij} = 1\right) \cap \left(c_{ij} = 1\right) \cap \left(d_{ij} = 1\right) \\ b_{ij}, \left(a_{ij} = 1\right) \cap \left(c_{ij} = 1\right) \cap \left(d_{ij} = 1\right) \\ \frac{U_j^{Up}}{U_i^{Up}}, \left(a_{ij} = 1\right) \cap \left(c_{ij} = 1\right) \cap \left(d_{ij} = 1\right) \cap \left(U_j^{\text{low}} = U_i^{\text{low}} = 0\right) \end{cases} \tag{28}$$

In Equation 28, it is observed that when the fuzzy attributes are equal (c_{ij} and d_{ij} are equal to the one unit), the effect of the p-factor disappears and the Camargo intrinsic cost function becomes independent of the type of t-norm intersection used (see Section 3.4.1. and Section 3.4.2.). In addition, the form of this function is analysed according to its derivative (Equation 28). Where the logical condition $\left(\left(\beta_i = 0\right) \cap \left(\beta_j = 1\right)\right)$ is associated with Supply Side Management Optimisation (Case 1), while the logical condition $(\beta_i = \beta_j = 0)$ is associated with Demand Side Management Optimisation (Case 2). In this way, Case 1 corresponds to a positive externality, implying a subsidy, whereas Case 2 corresponds to a negative externality, implying a tax or fine. This expression would be a good approximation since fuzzy attributes are usually close in value ($\mu_i \approx \mu_j$). Then the mathematical derivative of the Generic Camargo Intrinsic Cost depends on the inverse difference of the cost and emission weights ($\frac{1}{EW_j} - \frac{1}{EW_i}$). Depending on which one dominates, the Generic Camargo Intrinsic Cost curve will be increasing ($EW_j < EW_i$) or decreasing ($EW_j > EW_i$) and it will be constant ($EW_j = EW_i = 1$) for a set of exponential weights and preference functions (see Section 3.3.3.).

$$\left(\mu_i = \mu_j\right) \Rightarrow \left| GCIC_{ij} \right| = \left(-\frac{2 \cdot \beta_i - 1}{2 \cdot \beta_j - 1}\right) \cdot \left(\frac{EW_i}{EW_j}\right) \cdot \left(\frac{U_j^{Up} - U_j^{low}}{U_i^{Up} - U_i^{low}}\right) \cdot \mu_i^{\left(\frac{1}{EW_j} - \frac{1}{EW_i}\right)} \tag{29}$$

$$\frac{\partial}{\partial \mu_i} GCIC_{ij} = \begin{cases} > 0 & ,\left(\mu_i = \mu_j\right) \cap \left(\left(\left(\beta_i = \beta_j\right) \cap \left(EW_i > EW_j\right)\right) \cup \left(\left(\beta_i \neq \beta_j\right) \cap \left(EW_i < EW_j\right)\right)\right) \\ < 0 & ,\left(\mu_i = \mu_j\right) \cap \left(\left(\left(\beta_i = \beta_j\right) \cap \left(EW_i < EW_j\right)\right) \cup \left(\left(\beta_i \neq \beta_j\right) \cap \left(EW_i > EW_j\right)\right)\right) \\ ctte & ,\left(\mu_i = \mu_j\right) \cap \left(EW_i = EW_j\right) \end{cases} \tag{30}$$

If the attributes i and j have the same exponential weights $\left(EW_i = EW_j\right)$, which implies indifference with respect to the attributes (CO_2 and investment cost), then Equation 31 is obtained (see Section 3.4.1. and Section 3.4.2.).

$$\left(EW_i = EW_j\right) \Rightarrow \left| CIC_{ij} \right| = \left(\frac{U_j^{Up} - U_j^{low}}{U_i^{Up} - U_i^{low}}\right) \cdot \left(\frac{\mu_j}{\mu_i}\right)^{\frac{1}{EW_i}} \cdot \left(\frac{p + (1-p) \cdot \mu_j}{p + (1-p) \cdot \mu_i}\right) \tag{31}$$

If it is admitted that EW_i and EW_j are inverses, then Equation 32 is obtained (see Section 3.4.2.). This would imply that the decision maker has conflicting hierarchy and ranking criteria.

$$\left(EW_j = \frac{1}{EW_i}\right) \Rightarrow GCIC_{ij} = \left(-\frac{2 \cdot \beta_i - 1}{2 \cdot \beta_j - 1}\right) \left(\frac{1}{EW_j}\right)^2 \left(\frac{U_j^{Up} - U_j^{low}}{U_i^{Up} - U_i^{low}}\right) \left(\frac{\mu_i^{\left(1 - EW_j\right)}}{\mu_j^{\left(1 - \frac{1}{EW_j}\right)}}\right) \left(\frac{p + (1-p) \cdot \mu_j}{p + (1-p) \cdot \mu_i}\right) \left(\frac{\mu_j}{\mu_i}\right) \tag{32}$$

$$GCIC_{ij} = \left(-\frac{2 \cdot \beta_i - 1}{2 \cdot \beta_j - 1}\right) \cdot \left(\frac{1}{EW_j}\right)^2 \cdot \left(\frac{U_j^{Up} - U_j^{low}}{U_i^{Up} - U_i^{low}}\right) \cdot \left(\mu_i \cdot \mu_j^{\frac{1}{EW_j}}\right)^{\left(1 - EW_j\right)} \cdot \left(\frac{p + (1-p) \cdot \mu_j}{p + (1-p) \cdot \mu_i}\right) \cdot \left(\frac{\mu_j}{\mu_i}\right) \tag{33}$$

3.2.3. Influence of Exponential Weights (EW) on the attributes indices

An analysis of the preference function based on the influence of the exponential weights is then carried out (see Section 3.4.2). This analysis is important to understand the shape of the curves obtained and their practical implications, and for the sake of clarity and coherence of the development, it is analysed separately from the previous development. By developing Equation 2 and removing the index U_m associated with attribute m (CO_2 and investment cost), Equation 34 can be obtained.

$$U_m = \begin{cases} \left(U_m^{Up} - U_m^{Low}\right) \cdot \mu_m^{\frac{1}{EW_m}} + U_m^{Low} & ,(\beta_m = 0) \cap (0 \le \mu_m \le 1) \\ U_m^{Up} - \left(U_m^{Up} - U_m^{Low}\right) \cdot \mu_m^{\frac{1}{EW_m}} & ,(\beta_m = 1) \cap (0 \le \mu_m \le 1) \end{cases} \tag{34}$$

If the function is a decrease (and growth) function (It means $\beta_m = 0$ and $\beta_m = 1$), if the attribute has very high priority ($\frac{}{EW} \ll \mu$), its Exponential Weighted (EW_m) will be very high and therefore the preference function will be low ($\mu_m \ll 1$). In this way, in Equation 34 and Equation 35, if it is true that $\beta_m = 0$ (otherwise $\beta_m = 1$), and $\frac{1}{EW_m} \gg \mu_m$ (otherwise $\frac{1}{EW_m} \ll \mu_m$), then it occurs that $\mu_m^{\frac{1}{EW_m}} \to 0$ (otherwise $\mu_m^{\frac{1}{EW_m}} \to 1$).

Consequently, it follows that the index associated with the attribute U_m corresponds to the upper limit U_m^{Up} (it means $U_m \to U_m^{Up}$). Consequently, it follows that the index associated with the attribute U_m corresponds to the lower limit $\left(U_m \to U_m^{Low}\right)$. And vice versa, if it is true that $\beta_m = 0$ (otherwise $\beta_m = 1$), and $\frac{1}{EW_m} \ll \mu_m$ (otherwise $\frac{1}{EW_m} \gg \mu_m$), then it occurs that $\mu_m^{\frac{1}{EW_m}} \to 1$ (otherwise $\mu_m^{\frac{1}{EW_m}} \to 0$). Consequently, it follows that the index associated with the attribute U_m corresponds to the upper limit U_m^{Up} (it means $U_m \to U_m^{Up}$). This means that under these conditions, the lower extreme value of the fuzzy index is tended towards (see Section 3.4.2). This is the effect of the exponential weights associated with the decision maker hierarchy on the resulting indices, according to the present methodology, which will be demonstrated in the analysis of the optimisation results in the corresponding graphs. This will be demonstrated in Section 3.3.1 and in Section 3.3.2 and its relationship with the Generic Camargo Intrinsic Cost.

$$U_m = \begin{cases} U_m^{Low} & ,\left((\beta_m = 0) \cap \left((\mu_m)^{\frac{1}{EW_m}} \to 0\right)\right) \cup \left((\beta_m = 1) \cap \left((\mu_m)^{\frac{1}{EW_m}} \to 0\right)\right) \\ U_m^{Up} & ,\left((\beta_m = 0) \cap \left((\mu_m)^{\frac{1}{EW_m}} \to 1\right)\right) \cup \left((\beta_m = 1) \cap \left((\mu_m)^{\frac{1}{EW_m}} \to 1\right)\right) \end{cases} \tag{35}$$

3.3. Practical contributions of the proposed methodology: two cases

As explained in Section 3.1. of this paper, the methodology used is as follows:

Firstly, a life cycle analysis is carried out for the five sectors considered: resource extraction, material processing, manufacturing, construction and transport. This life cycle analysis takes into account the inputs of materials and fuels. Then, investment cost and emissions indices are obtained for the Argentine production chain according to two cases: 1) Supply Side Management (SSM) and 2) Demand Side Management (DSM). In Case 1, the aim is to minimise emissions by maximising the investment made. Case 2 seeks to minimise both indices or attributes.

Secondly, fuzzy functions are obtained from the investment and emission cost indices for each Exponential Weighting (EW) value. In addition, the fuzzy intersection is obtained for each type of t-norm according to the value of p.

Thirdly, these indices of the two cases are optimised using the Particle Swarm Optimisation metaheuristic.

Fourthly, these cases (Supply and Demand Management) are economically evaluated (carbon price) from the case of the fuzzy decision maker according to the optimal attributes of emissions and investment costs. Generic Camargo Intrinsic Cost Index (Section 3.2.) is used to economically evaluate the solutions obtained according to the two cases studied (Section 3.3.1. to Section 3.3.3.).

Fifthly, by analysing these indices, a new model for determining the novel Computable General Equilibrium Model is introduced (Section 3.4.) and the efficiency frontiers separating the feasible and non-feasible areas are obtained. This results in a new computable general equilibrium model in which the equilibrium points and feasible zones are searched and the marginal cost of equilibrium is obtained, as presented in Section 3.4. The methodology as a whole is novel and lays the groundwork for future research proposals in this area, including optimisation models and analysis.

The analysis starts with the investigation of Case 1, which refers to Supply Side Management (SSM) strategies $\left((\beta_i = 0) \cap (\beta_j = 1)\right)$. These strategies aim to increase investment costs (U_j) in order to reduce emissions (U_i) in the Argentine production chain, reflecting a positive externality and thus implying a subsidy (see Section 3.3.1.). After examining Case 1, the focus shifts to Case 2 ($\beta_i = \beta_j = 0$), which includes Demand Side Management (DSM) strategies and they aim to reduce investment costs (U_j) and by increasing production efficiency with minimal equipment and it is associated with Demand Side Management Optimisation (see Section 3.3.2.) and the emissions (U_i). It indicates a negative externality and therefore implies a tax or a fine.

After discussing Case 1 and Case 2 separately, the analysis compares the results of these strategies, including an examination of the Generic Camargo Intrinsic Cost (GCIC) in both cases, which provides insight into the economic implications of each approach. Throughout the analysis, Argentine government and international market data on CO_2 emissions are used to calibrate and validate the model (Argentina, 2023; Spain, 2023). Curves generated from Particle Swarm Optimisation solutions for different exponential weights (EW) are used to illustrate the results of the strategies. The analyses are first presented separately for each exponential weight to validate the calculated curves and compare them with international carbon prices. The findings and contributions of this work are then outlined, including their potential implications for future computational general equilibrium models of projects (Section 3.4.1.). Complementary analysis is also provided to further elucidate the implications of the policies discussed (Section 3.4.2.). Overall, these efforts are in line with the overall objective of this paper, which is to compare novel economic valuation models in order to establish an emissions price for valuing externalities and providing effective environmental regulatory tools.

3.3.1. Case 1: Supply Side Management Optimisation

First, in this subsection, Supply Side Management Optimisation aims to minimise emissions while maximising investment costs in the Argentinean production chain $\left((\beta_i = 0) \cap (\beta_j = 1)\right)$. This is a case of a positive externality from the production chain to demand, as the investment benefits society by reducing emissions. Figure 3 and Figure 4 illustrate the impact of different exponential weights, where Investment costs in $\left[\frac{USD}{MWh}\right]$ and emissions in $\left[\frac{Ton\,CO_2\,EQ}{Mwh}\right]$ are in line with the data provided by the Ministry of Energy and international values (Argentina, 2023; Spain, 2023).

The life cycle analysis yielded an emission interval of $[0.30; 0.39]\frac{Ton\,CO_2\,EQ}{Mwh}$ and the investment costs yielded an emission interval of $[1.5; 5]\frac{USD}{Mwh}$, the limits of which correspond to the fuzzy limits (see Figure 1, Figure 3 and Figure 4). The case where the exponential weights are reciprocal ($EW_j = \frac{1}{EW_i}$) was analysed, which makes their relationship in the Generic Camargo Intrinsic Cost exponential (see Equations 29 to 32). This observation shows that the cost vs. emissions curves are increasing and compact within a feasible cost vs. emissions combination zone for the proposed efficiency.

Secondly, as the priority (exponential weight) associated with the emissions (U_i) decreases, the value of the exponential weight decreases accordingly ($EW_i \to 0$). Consequently, when the reciprocal exponential weights are evaluated ($EW_j = \frac{1}{EW_i}$), if the emission is considered unimportant and its exponential weight is small ($EW_j \ll 1$), then the investment cost (U_j) becomes significant, leading to a high exponential weight ($EW_j \to \infty$). Consequently, it is observed that the curve tends to be horizontal and linear towards the upper cost limit (U_j^{Up}). Conversely, as the priority of emissions increases ($EW_i \to \infty$), the curve tends to be vertical and linear towards the lower cost limit. This observation is consistent with the mathematical analysis carried out in Section 3.2.3. With this in mind, when the fuzzy function increases ($\beta_m = 1$) and the attribute has a very high priority ($EW_m \gg 1$), its preference function will be very low ($\mu_m \ll 1$). Conversely, when the function is decreasing (Equation 34 and Equation 35), if the attribute has a very low priority ($EW_m \ll 1$), its preference function will be very low ($\mu_m \gg$). Then the function will tend to the upper (U_m^{Up}) or lower (U_m^{Low}) value according to the extreme cases seen in this analysis. The function will tend to the upper cost (U_j^{Up}) when it happens that $\left(EW_j \to \infty\right) \cap \left(EW_i \to 0\right)$.

Thirdly, the figures are presented in order of t-norm type: $p = 0$ (Figure 3a), $p = 1$ (Figure 3b), $p = 2$ (Figure 4a) and $p = 3$ (Figure 4b). It can be observed that the curves are concentrated in a region (feasible region) whose area is maximum for $p = 0$ (Figure 3a) and minimum for $p = 2$ in Figure 3a, and that this area decreases as p increases and vice versa, indicating that the non-feasible region increases (see section 3.4). As a result, the Einstein product, which is the most demanding (the lowest t-norm function $tp\left(\mu_i, \mu_j\right)$), has the smallest feasible area. This phenomenon

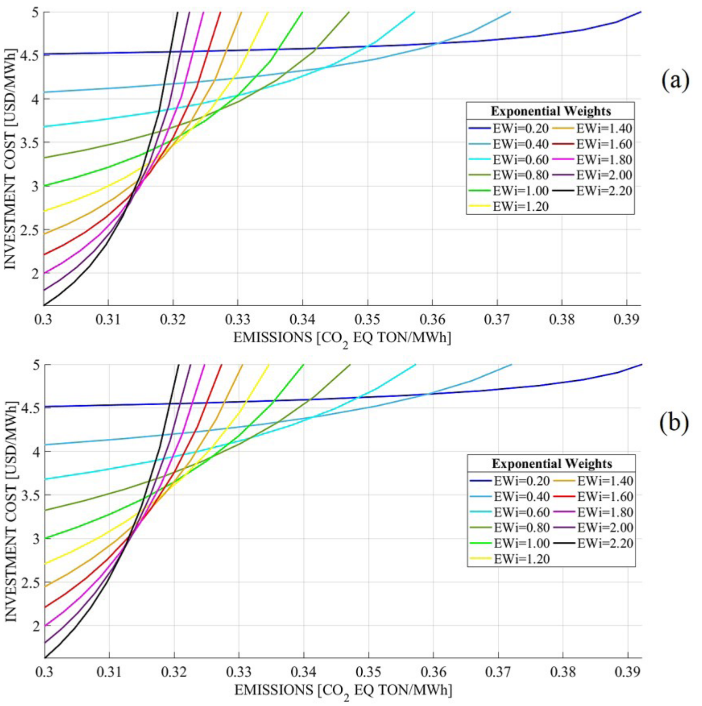

Figure 3. Investment Cost (U_j) vs. Emissions (U_i) for: (a) Particular Hamacher Product $(p=0)$ and (b) Generic Hamacher Product $(p=0.5)$ in Case 1.
Source: The authors.

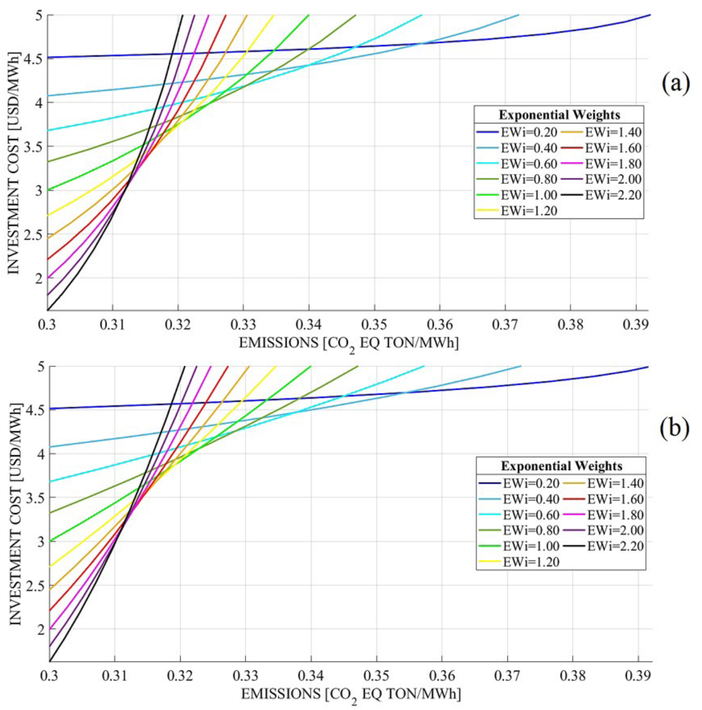

Figure 4. Investment Cost (U_j) vs. Emissions (U_i) for: (a) Algebraic Product $(p=1)$ and (b) Einstein Product $(p=2)$ in Case 1.
Source: The authors.

is reminiscent of the supply curve in economic theory and regulation, a concept that will be discussed further in Section 3.4. It can therefore be concluded that, similar to the supply curve in economic theory, the present proposal for Supply Side Management results in a curve from which the production chain tries to identify the boundary between the feasible and non-feasible zones. The results are logical and, in principle, the curves calculated by the PSO are correct. It should be noted that the slope of the investment cost curves also changes as the factor p varies, and this in turn affects the value of the Generic Camargo Intrinsic Cost Index, as will be seen in Section 3.3.3.

3.3.2. Case 2: Demand Side Management optimisation

Figures 5 and 6 show the Demand Side Management of optimal investment costs and emissions.

Firstly, in this subsection, Demand Side Management (DSM) optimisation aims to minimise emissions while minimising investment costs in the Argentinean production chain ($\beta_i = \beta_j = 0$), where Figure 5 and Figure 6 illustrate the impact of exponential weights resulting from changes in their priorities. This is a case of a negative externality from the production chain to the demand side, as the minimum investment harms society by reducing emissions less. In addition, the analysis includes the case where exponential weights are reciprocal ($EW_j = \frac{1}{EW_i}$). Investment costs in $\left[\frac{USD}{MWh}\right]$ and emissions in $\left[\frac{\text{Ton CO}_2\text{ EQ}}{\text{Mwh}}\right]$ are in line with the data provided by the Ministry of Energy and international values (Argentina, 2023; Spain, 2023). The life cycle analysis yielded an emission interval of $[0.30;0.39]\frac{\text{Ton CO}_2\text{ EQ}}{\text{Mwh}}$ and the investment costs yielded an emission interval of $[1.5;5]\frac{\text{USD}}{\text{Mwh}}$, the limits of which correspond to the fuzzy limits (see Figure 1, Figure 5 and Figure 6).

Secondly, similar to Case 1, as the priority (exponential weight) associated with the emissions (U_i) decreases, the value of the exponential weight decreases accordingly ($EW_i \to 0$). Consequently, when the reciprocal exponential weights are evaluated ($EW_j = \frac{1}{EW_i}$), if the emission is considered unimportant and its exponential weight is small ($EW_j \ll 1$), then the investment cost (U_j) becomes significant, resulting in a high exponential weight ($EW_j \to \infty$). Consequently, it is observed that the curve tends to be horizontal and linear towards the lower cost limit (U_j^{Low}). Conversely, as the priority of emissions increases ($EW_i \to \infty$), the curve tends to be vertical and linear towards the upper cost limit (U_j^{Up}).

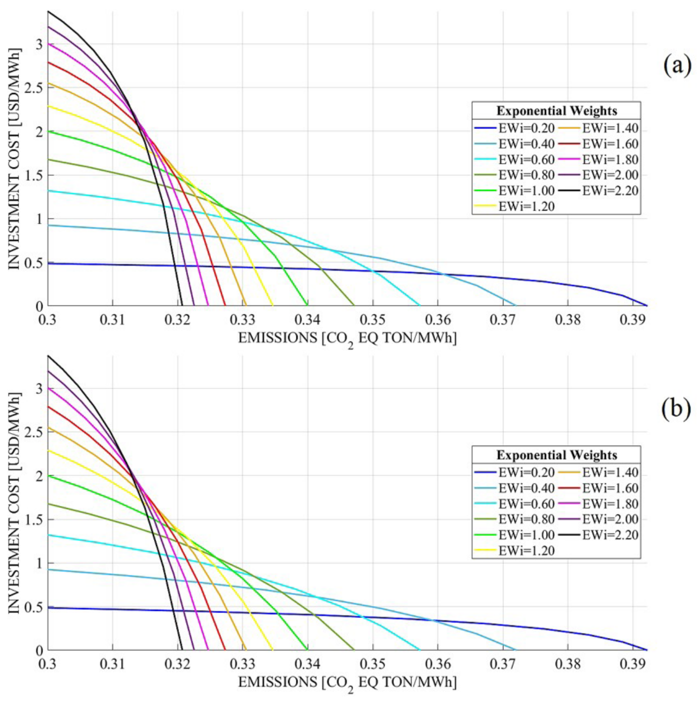

Figure 5. Investment Cost (U_j) vs. Emissions (U_i) for: (a) Particular Hamacher Product ($p = 0$) and (b) Generic Hamacher product ($p = 0.5$) in Case 2.
Source: The authors.

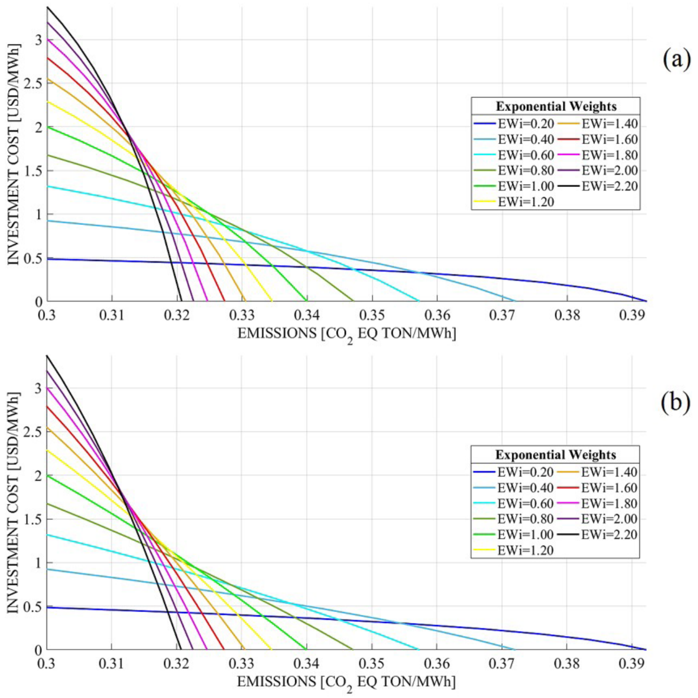

Figure 6. Investment Cost (U_j) vs. Emissions (U_i) in Case 2 for: (a) $(p=1)$ and (b) $(p=2)$.
Source: The authors.

This observation is consistent with the mathematical analysis carried out in Section 3.2.3. In this way, if the function is increasing ($\beta_m = 1$) and the attribute has a very high priority ($EW_j \gg 1$), its preference function will be very low ($\mu_m \ll 1$). Conversely, when the function is decreasing (Equation 34 and Equation 35), if the attribute has a very low priority ($EW_j \ll 1$), its preference function will be very high ($\mu_m \gg 1$). The function will tend to the upper (U_m^{Up}) or lower (U_m^{Low}) value according to the extreme cases seen in this analysis. In this case, the function will tend to the upper cost (U_j^{Up}) limit when it happens that $\left(EW_j \to 0\right) \cap \left(EW_i \to 0\right)$. It can be seen that the curves are concentrated in a zone (feasible zone) that is maximum for $p=0$ and decreases as p increases, i.e. the non-feasible zone expands. Therefore, the one with the smallest feasible area is the Einstein product, which is the most demanding. This is reminiscent of the demand curve in economic theory and regulation, and is explained in Section 3.4.

Thirdly, similar to the first case, the figures are presented in order of t-norm type: $p=0$ (Figure 5a), $p=0.5$ (Figure 5b), $p=1$ (Figure 6a) and $p=2$ (Figure 6b). The curves are concentrated in a region (feasible region) whose area is maximum for $p=0$ (Figure 3a) and minimum for $p=2$ in Figure 3a, and that this area decreases as p increases and vice versa, indicating that the infeasible region increases (see Section 3.4).

A feasible region is then bounded by the maximum desired investment cost and the minimum desired efficiency of the proposed solution. All solutions within the infeasible region (defined by the set of curves for each exponential weight that make up the infeasible region) would yield less than the expected efficiency (tp_{boundary}). The boundary between the feasible and infeasible regions is known as the efficiency frontier or Pareto frontier of the maximum tolerable investment (Section 3.4.2.). It can be observed that, in line with the supply curve in economic theory, the present Demand Side Management proposal yields a curve from which the demand of the production chain seeks the boundary between the feasible and non-feasible zones, as these points represent where production costs are highest and emissions are minimised.

Fourthly, the intersection of the curves from the two cases (Case 1 and Case 2) gives the feasible solution, similar to the producer surplus in the law of supply and demand. Consequently, if emissions are negligible, the exponential weight will be small and the preference function will be greatly diluted (see Figure 1). Therefore, the investment made will be indifferent and, as a consequence, it will not vary, tending towards the value of the index in its fuzzy transition towards its upper limit U_j^{Up} for any emission value (see Equation 2 with $\beta_j = 0$ and Figure 2). Similarly to Case 1, it is observed that the curves are concentrated in a region (feasible region) that is maximal for $p=0$ and decreases as p increases, indicating an expanding non-feasible region. Consequently, the Einstein product, which is the most demanding (the lowest t-norm function $tp(\mu_i, \mu_j)$), has the smallest feasible area. This phenomenon is similar to the supply curve in economic theory and regulation, a concept that will be discussed further in Section 3.4. It can therefore be concluded that, like the supply curve in economic theory, the proposed approach to supply side management results in a curve from which the production chain seeks to identify the boundary between the feasible and non-feasible zones. This boundary represents the points where production costs are minimised and emissions are minimised. The Generic Camargo Intrinsic Cost can be negative according to the mathematical development, but its absolute value is taken into account. The results are logical and, in principle, the curves calculated by the PSO are correct. It should be noted that the slope of the investment cost curves also changes as the factor p varies, and this in turn affects the value of the Generic Camargo Intrinsic Cost Index, as will be seen in Section 3.3.3.

3.3.3. Comparison of intrinsic cost (case 1 and case 2)

This section compares and analyses the two cases (Case 1 and Case 2), focusing on the influence of these exponential weights (associated with the decision maker's priorities) on the resulting indices (Figure 7 and Figure 8). These figures are presented in order of t-norm type: $=0$ (Figure 7a), $p=1$ (Figure 7b), $p=2$ (Figure 8a) and $p=3$ (Figure 8b).

The impact of the resulting fuzzy confluence variation is shown in Figure 7 and Figure 8, which illustrate the analysis of two scenarios within the Argentinean production chain: 1) Supply Side Management (SSM) and 2) Demand Side Management (DSM) optimisation, using different Exponential Weights (EW) as determined by the Analytical Hierarchy Process. In the case of a positive externality, the Generic Camargo Intrinsic Cost (GCIC) will be positive (according to the mathematics developed), while the opposite is true for a negative externality; however, in the present analysis, the absolute value is considered. This approach simplifies the analysis as both indices have the same magnitude but different signs. Nevertheless, the sign is interpreted in the analysis to indicate the effect that the increase or decrease in emissions will have from the case being analysed (Supply and Demand Side Management).

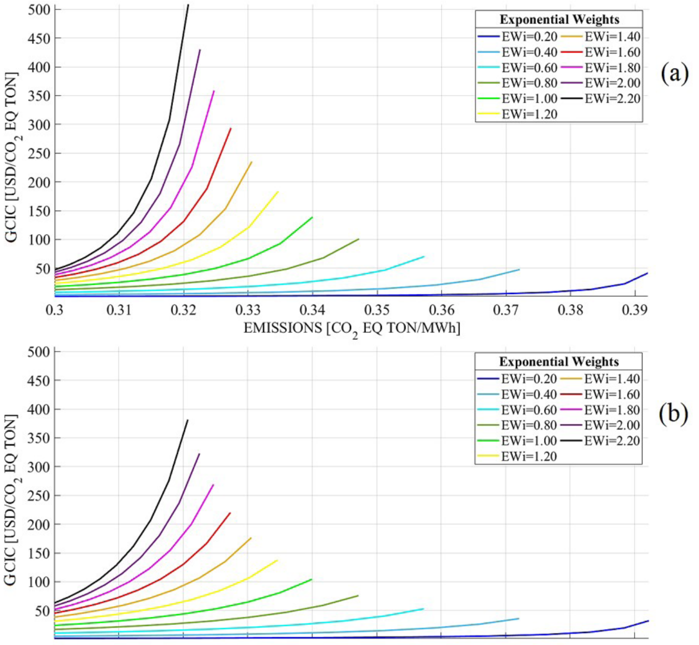

Figure 7. Generic Camargo Intrinsic Cost vs. Emissions (U_i) for both cases (1 and 2): (a) Particular Hamacher Product (p=0) and (b) Generic Hamacher Product (p=0.5).

Source: The authors.

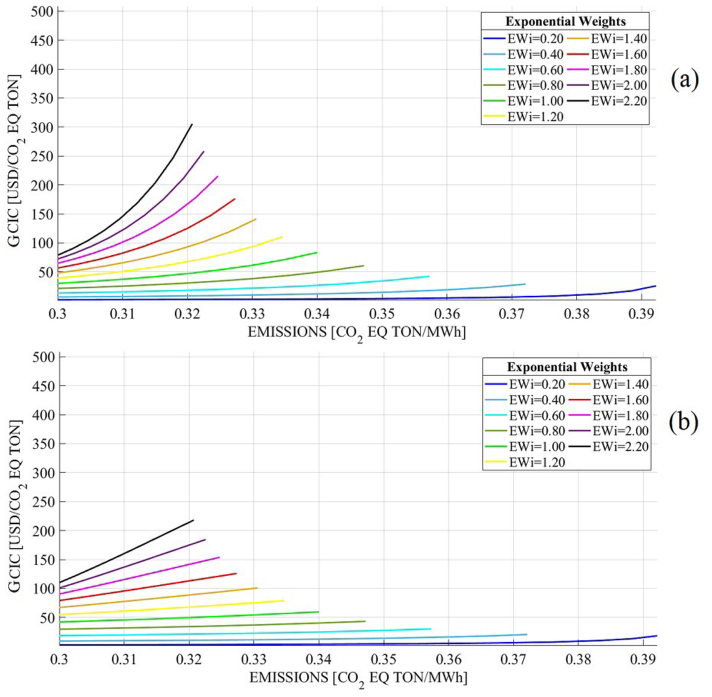

Figure 8. Generic Camargo Intrinsic Cost vs. Emissions (U_i) for: (a) Algebraic Product $(p=1)$ and (b) Einstein Product $(p=2)$ for both Cases (1 and 2).
Source: The authors.

Firstly, the graph of the two calculated attributes (investment cost U_j and emissions U_i) is examined in relation to the analysed attributes and the Generic Camargo Intrinsic Costs. Figure 7 and Figure 8 show an emission interval $[0.30;0.39]\frac{\text{Ton CO}_2\text{ EQ}}{\text{Mwh}}$, which is consistent with the results in the above sections. The interval for the Generic Camargo Intrinsic Cost is $[0;500]\frac{USD}{Ton\ CO_2\ EQ}$, reaching $500\frac{USD}{Ton\ CO_2\ EQ}$ in extreme solutions, while historical carbon bond prices are in the range $[20;100]\frac{USD}{Ton\ CO_2\ EQ}$ (see Figure 7).

Secondly, it is important to analyse that any reduction in emissions represents a positive externality (Case 1), whereas any increase in emissions represents a negative externality (Case 2). Similarly to the first case, the figures are presented in order of t-norm type: $p=0$ (Figure 7a), $p=0.5$ (Figure 7b), $p=1$ (Figure 8a) and $=2$ (Figure 8b). It is observed that the slope of the generic Camargo intrinsic cost is inversely related to the factor p, which defines the family of t-norm Hamacher's products. Specifically, when $p=0$ (particular Hamacher's product), these intrinsic cost curves have high slopes, whereas for high values of p $(p=2)$, the Generic Camargo intrinsic cost curve has low slopes. This information is crucial when defining the externality; choosing low values of p can lead to significant variation in the penalty or subsidy amount, while the opposite is true for high values of p.

Thirdly, a feasible region is defined based on the maximum desired efficiency of the proposed solution. It can be observed that the curves are concentrated in an area (feasible area) whose area is maximal for $p=0$ (Figure 7a) and minimal for $p=2$ in Figure 8a and this area decreases as p increases, and vice versa, indicating that the non-feasible area is increasing (see Section 3.4). Then Consequently, the Einstein product, which is the most demanding (the lowest t-norm function $tp(\mu_i,\mu_j)$), has the smallest feasible area. Solutions falling within the infeasible region (defined by the curves representing each exponential weight) would deliver less than the expected efficiency (tp_{min}). The boundary between the feasible and infeasible regions forms the efficiency frontier or Pareto frontier of the maximum tolerable investment. The intersection of the two curves indicates the feasible solution, analogous to the producer surplus in the law of supply and demand. Note that the endpoints of these curves form a "U" curve that defines a feasible region (see Section 3.4.2). This feasible region is defined by the set of possible values of these curves (the image of the function), while an infeasible region is defined by the values that cannot be reached for any exponential weighting, unless the efficiency of the required solution is reduced (value of the t-norm). The results are consistent and allow comparisons between different curves, as presented in Section 3.3.3.

Fourthly, and as a result, the Generic Camargo Intrinsic Cost remains a reliable indicator of energy efficiency and sustainability, as it includes both economic costs and environmental costs or benefits (CO_2 emissions), while the price of carbon bonds facilitates investment in the Argentinean production chain. By refining the mathematical equation of the Generic Camargo Intrinsic Cost Index, this work has successfully extended its applicability to other types of t-norms (continuous and differentiable). The logical results show that the curves calculated by the Particle Swarm Optimisation are generally accurate and the prices obtained are in line with international values (Camargo, 2019, 2021, 2022a, b, 2023; Argentina, 2023; Spain, 2023).

3.4. Economic contributions of the proposed methodology

3.4.1. Generic Camargo Intrinsic Cost analysis with equal fuzzy functions

Firstly, in this section, the two cases in the Argentinean production chain are analysed in relation to the Generic Camargo Intrinsic Cost analysis with equal fuzzy functions, highlighting the theoretical, practical and economic contributions in the Argentinean production chain. The Figure 9 shows an emissions interval $\left[0.30;0.39\right]\frac{\text{Ton } CO_2 \text{ EQ}}{\text{Mwh}}$, which is consistent with the emissions reported in the above sections.

Secondly, Figure 9 shows the effect of varying the exponential weights (EW) as a result of varying their priorities when the exponential weights are reciprocal ($\mu_i = \mu_j$), which would make the Generic Camargo Intrinsic Cost exponential (see Equation 29 in Section 3.2.2). In this situation, since the preference functions are the same ($\mu_i = \mu_j$), the Generic Camargo Intrinsic Cost is indifferent to the t-norm used and therefore the same curve is obtained regardless of the value of p. In addition, and in the same way as discussed above, the exponential weights associated with investment cost and emissions are reciprocal ($EW_j = \frac{1}{EW_i}$). With this in mind, the results of this analysis (Figure 9) are interesting.

To do this, it is necessary to revisit the analysis presented in Equation 29 and Equation 30 of Section 3.2.2, where the influence of exponential weights ($EW_{i/j}$) and preference functions ($\mu_{i/j}$) on the Generic Camargo Intrinsic Cost was discussed. In this sense, it was seen that the mathematical derivative of the Generic Camargo Intrinsic Cost depends on the inverse difference of the cost and emission weights ($\frac{1}{EW_j} - \frac{1}{EW_i}$). Depending on which one dominates, the Generic Camargo Intrinsic Cost curve will be increasing ($EW_j < EW_i$) or decreasing ($EW_j > EW_i$) and it will be constant ($EW_j = EW_i = 1$) for a set of exponential weights and preference functions. In the case that the Generic Camargo Intrinsic Cost is constant, its value will correspond to the objective valuation given by the maximum and minimum limits of emissions and costs obtained.

Thirdly, these curves have a feasible and infeasible range for the efficiency analysed, which is also discussed in more detail in the next subsection. All solutions in the feasible region (determined by the set of curves for each exponential weight that make up the infeasible region) would require more than the expected efficiency (tp_{min}), as explained in section 3.4.2. The Generic Camargo Intrinsic Cost (GCIC) obtained is consistent and logical with the information provided by the Argentinean Chamber of Renewable Energy and the Ministry of Energy, as well as with international values (Argentina, 2023; Spain, 2023). In this way, the boundary between the

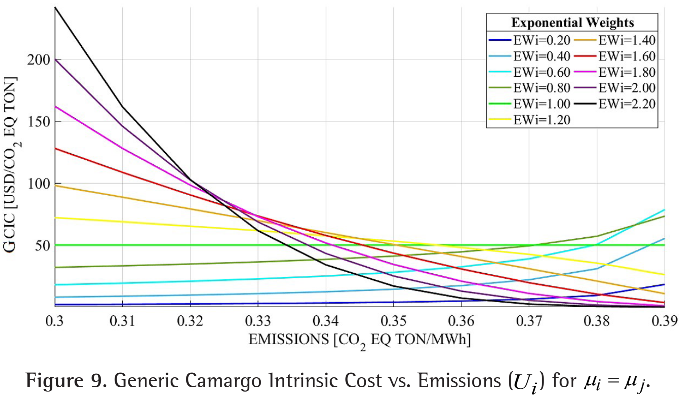

Figure 9. Generic Camargo Intrinsic Cost vs. Emissions (U_i) for $\mu_i = \mu_j$.
Source: The authors.

feasible and non-feasible zones forms a concave 'U'. This has the same characteristic as the marginal cost curve and therefore shows that the intrinsic cost effectively models the marginal cost of the variable being analysed.

3.4.2. Infeasible region and introduction to a Computable General Equilibrium Model (CGEM)

Firstly, in this section, a feasible region is observed, which is delineated by the minimum desired efficiency of the proposed solution (Figure 10a, Figure 10b, Figure 11a and Figure 11b). Solutions within the infeasible range would imply less than the expected efficiency ($tp_{boundary}$). The feasible region is defined by the set of curves for each exponential weight. The boundary between the feasible and infeasible regions is defined by the efficiency frontier or Pareto frontier of the maximum (Equation 36) and minimum (Equation 37) tolerable investment costs. In Case 1 (Supply Side Management optimisation), this boundary represents the efficiency frontier or Pareto frontier of the minimum tolerable investments (Equation 36 and Figure 10a). Similarly, in Case 2 (Demand Side Management optimisation), the boundary ($U_{boundary}$) between the feasible and infeasible regions corresponds to the efficiency frontier or Pareto frontier of the maximum tolerable investments (Equation 37 and Figure 10b). Case 1 $\left((\beta_i = 0) \cap (\beta_j = 1)\right)$ is associated with Supply Side Management Optimisation (Equation 36), while Case 2 ($\beta_i = \beta_j = 0$) is associated with Demand Side Management Optimisation (Equation 37). Recall that case 1 corresponds to a positive externality and therefore implies a subsidy, whereas case 2 corresponds to a negative externality and therefore implies a tax or a fine (Figure 11a and Figure 11b).

$$U_j \geq U_{boundary} \quad \forall \quad \left(tp\left(\mu_i, \mu_j\right) \geq tp_{boundary}\right) \cap \left(\left(\beta_i = 0\right) \cap \left(\beta_j = 1\right)\right) \tag{36}$$

$$U_j \leq U_{boundary} \quad \forall \quad \left(tp\left(\mu_i, \mu_j\right) \geq tp_{boundary}\right) \cap \left(\beta i = \beta_j = 0\right) \tag{37}$$

Secondly, it should be remembered that Equation 38 takes into account the sign resulting from the consideration of the two optimisation directions and the one corresponding to the subjective evaluation developed in Section 3.2 (Generic Camargo Intrinsic Cost), Section 3.3.1. (Supply Side Management), Section 3.3.2. (Demand Side Management) and Section 3.3.3.

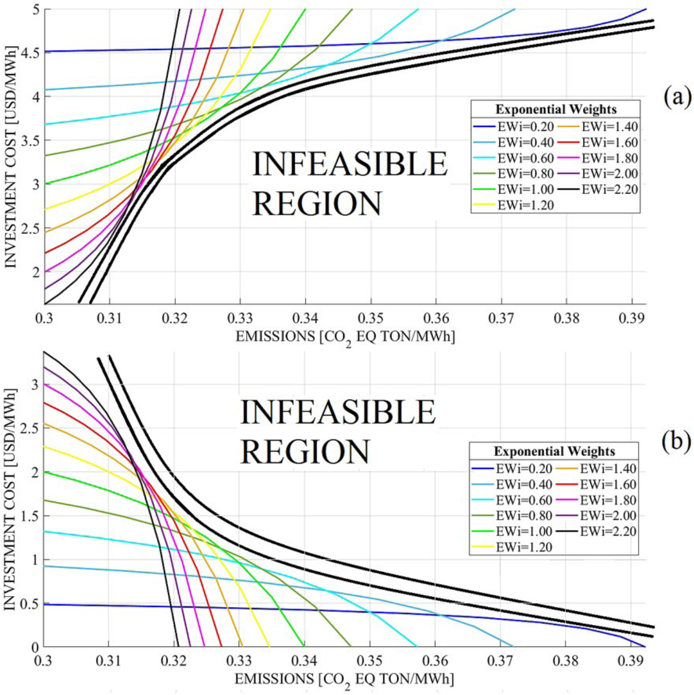

Figure 10. Investment Cost (U_j) vs. Emissions (U_i) with $EW_j = 1/EW_i$, $p = 0$ and the following situations: (a) Case 1 and (b) Case 2.
Source: The authors.

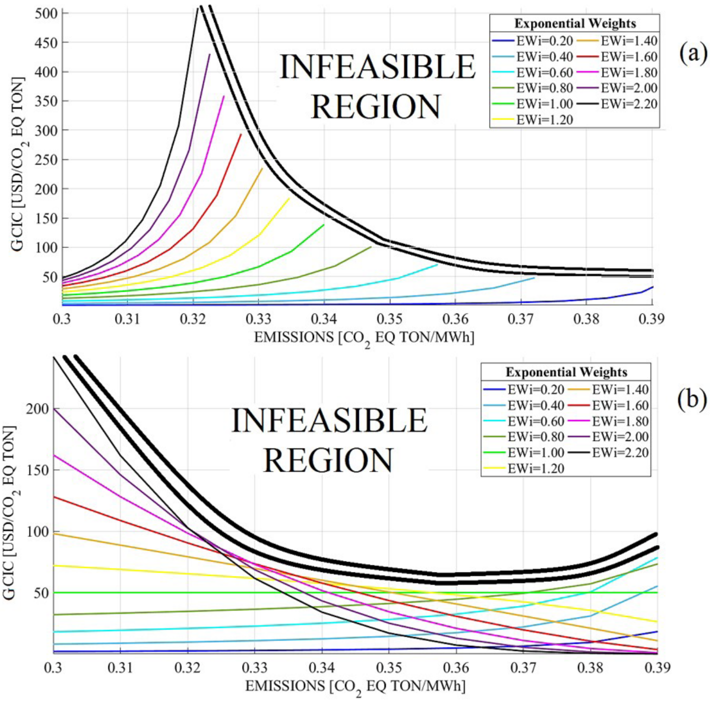

Figure 11. Generic Camargo Intrinsic Cost vs. Emissions (U_i) with $p = 0$ and the following situations: (a) $EW_j = 1 / EW_i$ and (b) $\mu_i = \mu_j$.
Source: The authors.

Equations 38 and 39 take into account the sign resulting from the consideration of the two optimisation directions and the one corresponding to the investment cost limit for the two cases: 1) Supply Side Management (Section 3.3.1.) and 2) Demand Side Management (Section 3.3.2.). Equation 38 and Equation 39 show the Generic Camargo Intrinsic Cost limits according to Case 1 (Equation 38) and Case 2 (Equation 39).

$$(GCIC_{i\,j} \ge GCIC_{boundary}) \ \forall \ \left(tp \ge tp_{boundary}\right) \cap \left(\beta_i = \beta_j\right) \tag{38}$$

$$\left(GCIC_{i\,j} \le GCIC_{boundary}\right) \ \forall \ \left(tp \ge tp_{boundary}\right) \cap \left(\left(\beta_i = 0\right) \cap \left(\beta_j = 1\right)\right) \tag{39}$$

In this way, a feasible region is observed in relation to the GCIC curves, bounded by the minimum desired efficiency ($tp_{boundary}$) of the proposed solution. The feasible region is determined by the set of curves for each exponential weight (Figure 11a and Figure 11b). In Case 1 (Supply Side Management optimisation), the boundary ($GCIC_{boundary}$) between the feasible and infeasible regions corresponds to the efficiency frontier or Pareto frontier of the maximum tolerable investments (Equation 39 and Figure 11b). Similarly, in Case 2 (Demand Management Optimisation), this boundary represents the efficiency frontier or Pareto frontier of the maximum tolerable investments (Equation 38 and Figure 11a and Figure 11b).

Thirdly and additionally, a break-even point (see Figure 12a) of $0.311 \text{ Ton } CO_2 \text{ EQ}$ and $2.5 \frac{USD}{MWh}$ was obtained and the break-even zone obtained where Emissions (U_i) are between $\left[0.3; 0.311\right] \frac{Ton\ CO2\ EQ}{MWh}$ and the Investment Cost (U_j) are between $\left[1.6; 3.4\right] \frac{USD}{MWh}$ (see Figure 12a and Figure 12b). This equilibrium analysis was only carried out for the case of the Hamacher T-Norm product, which was chosen as the least demanding and used as a reference point. Given the exhaustive nature and broad scope of the comparison with different T-norms, it was not considered practical to fully develop this analysis within this paper and therefore warrants further exploration in future studies.

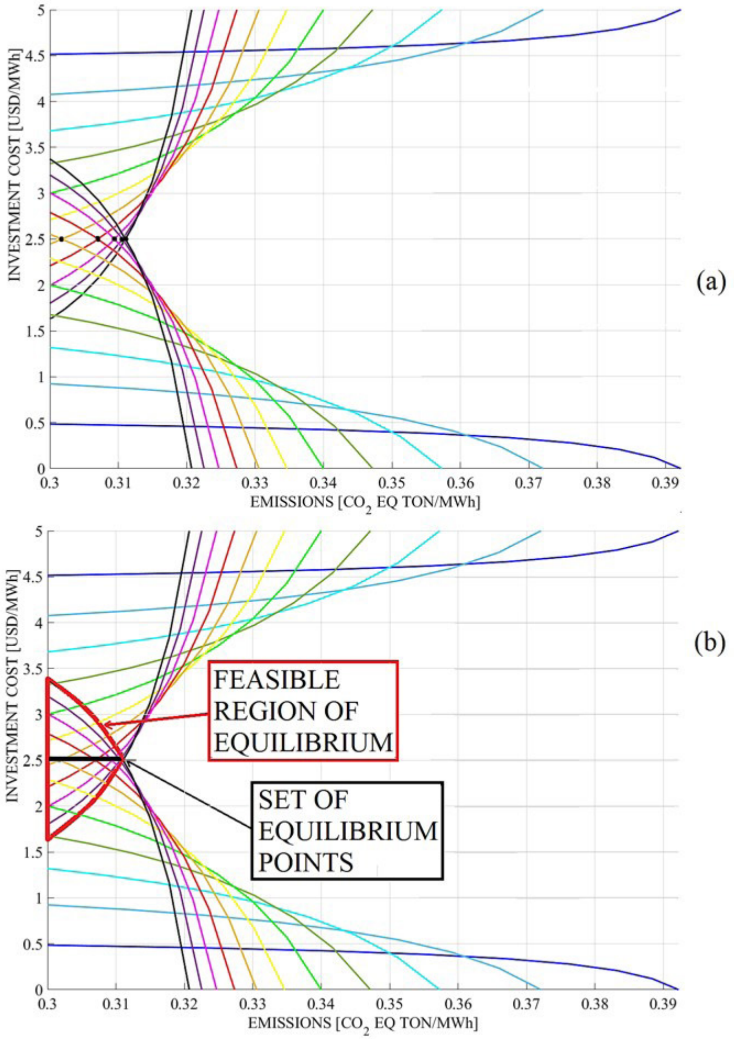

Figure 12. Feasible Region of equilibrium for Investment Cost vs. Emissions ($p = 0$): (a) Points of equilibrium and (b) Zones of equilibrium.
Source: The authors.

Finally, these results show that fuzzy decision theory can delineate a zone and an equilibrium point depending on the hierarchy of agents (see Figure 12a and Figure 12b).

This novel methodology represents a significant advance in computable general equilibrium models, allowing the development of models that derive equilibrium supply and demand points from the hierarchy (and vice versa) with uncertainty (see Section 2.4). In particular, this can be achieved with variables with or without associated markets and with known or unknown prices, which is a notable advantage. Future work will deepen this analysis and proposal, including current t-norms and potential state-of-the-art advances. This boundary between the feasible and infeasible regions is then defined by the efficiency frontier (Equation 38 and Equation 39), or Pareto frontier, of the maximum feasible solution.

4. Conclusions

The current line of research has presented a methodology to solve the following challenges in the economic evaluation of optimal production chains, which were highlighted in the introduction and are still discussed in the state of the art (Camargo, 2019, 2021, 2022a, b, 2023; Camargo et al., 2018, 2023a, b): 1) The challenges of multidisciplinary or multi-objective optimisation in the presence of fundamental uncertainty. 2) The complexity of mathematical modelling due to the previous point, especially in terms of metric compatibility (equality of units of measurement) of the optimal indices and their hierarchical evaluation. Thus, if one of the indices corresponds to an economic value and the other is a non-economic technical index, this implies an economic evaluation of this non-monetisable index, which may be unknown and not easy to obtain. 3) Due to the previous points, there is an inefficiency in determining the optimal economic valuation (price of CO_2 and externality), especially when there are indices that cannot be monetised or are not subject to the laws of supply and demand on the markets. 4) Inadequate consideration of fundamental uncertainties in the current state of the art due to the difficulty or lack of consensus in the optimal economic valuation of non-monetisable indices of the corresponding externality (positive or negative). 5) The best use of the aforementioned artificial intelligence tools to obtain and evaluate optimal solutions in this context.

Based on the above five state-of-the-art problems, this novel methodology aimed to obtain the optimal economic valuation of emissions (carbon price) under uncertainty (Fuzzy Decision Making) and hierarchical variation (Analytic Hierarchy Process) within the Argentine production chain (Life Cycle Analysis), resulting in a novel model of market equilibrium. The originality aspects of this methodology included: 1) the development of a novel optimal economic (marginal) evaluation index called Generic Camargo Intrinsic Cost for three fuzzy intersection operators of Hamacher families of fuzzy intersection operators with changing hierarchy, 2) the determination of optimal graphical attribute efficiency points, regions and boundaries along with their optimal economic evaluation, and 3) the creation of a computable general equilibrium model with fundamental uncertainty. The research method involved the theoretical, practical and economic contribution and results (including mathematical and graphical analysis) of this methodology and its novel tools, which were developed, generalised and analysed.

These tools, improved, developed and combined by the present novel methodology, are based on bio-inspired algorithms, heuristics and metaheuristics for searching and solving bio-inspired problems and emulating human behaviour in making and weighing (hierarchical) decisions according to artificial intelligence tools to find the optimal economic evaluation.

Therefore, the following methodology was used (see Figure 2).

Firstly, a Life Cycle Assessment (LCA) was carried out, considering the material and fuel flow of the Argentine Production Chain (APC) through the following stages: resource extraction, material processing, manufacturing, construction, transportation and waste management (Figure 2). In this way, the parameters (including technical data) were mainly processed and the complete model was validated using information from public reports available in the database of the Ministry of Energy and Mines (Camargo, 2019, 2021, 2022a, b, 2023; Argentina, 2023; Camargo et al., 2023a, b). Two cases have been considered in this optimisation: Supply Side Management (SSM) and Demand Side Management (DSM). Supply Side Management (SSM) efficiency optimisation involved reducing emissions by investing in energy efficient improvements in the production chain or by changing consumption patterns with minimal investment. Demand Side Management (DSM) efficiency improvements involved reducing investment and emissions by implementing measures that encourage conservation.

Secondly, based on the Life Cycle Analysis (LCA) model, the efficiency index to be optimised (fuzzy intersection) was determined and the attributes to be evaluated (investment costs and CO_2 emissions) were defined. The decision maker (Figure 2) then evaluated the indices (emissions and investment costs) resulting from the Life Cycle Analysis and transformed them into the fuzzy domain, taking into account the upper and lower bounds (see Equation 1) obtained from the upper and lower limits of the analysed Life Cycle Analysis (U_m^{Up} and U_m^{Low}). This methodology combined this model with fuzzy decision theory and the Analytic Hierarchy Process to obtain the fuzzy indices, which incorporate the upper and lower bounds obtained from the life cycle analysis, the hierarchical evaluation (exponential weights) and the type of fuzzy intersection operator to be used according to the p-coefficient used. The exponential weights (EW) represented the decision maker hierarchy (Supply and Demand Side Management) for the evaluated attributes (investment cost and emissions), influencing the optimal solution and consequently the economic valuation (carbon tax or carbon price) based on the assigned hierarchy.

Thirdly, to find the optimal solution, this methodology used fuzzy decision theory and analytic hierarchy process with artificial intelligence tools such as particle swarm optimisation metaheuristics. It provided optimal solutions for supply chain management and industrial processes. The values of objectives and constraints depended on the solution proposed by the Particle Swarm Optimisation (PSO) metaheuristic and the prioritisation proposed by the Analytic Hierarchy Process. In addition, the hierarchy (Analytic Hierarchy Process) and the upper and lower bounds (static) of these attributes were determined. From there, the evaluation of the indices was carried out, which also depended on the direction of improvement of the function (increase or decrease). The two cases (supply and demand management optimisation) were analysed for the reduction of environmental impacts (emissions), where these cases were made according to the decision maker's cases on emissions and the cost of the investment (carbon price). In this sense, this metaheuristic aimed to maximise both the fuzzy indices and their intersection, regardless of whether they were at maximum or minimum attribute values (investment cost and emissions). If the aim is to maximise the given indices then the value of the fuzzy upper limit (U_m^{Up}) will be sought to be reached. Conversely, if the aim is to minimise the indices then the value of the fuzzy lower limit (U_m^{Low}) will be sought to be reached (see Equation 1).

Fourthly, the Generic Camargo Intrinsic Cost that model the economic valuation (carbon price) of CO_2 emissions from the Argentinean production chain was obtained and compared, considering both Supply Side Management and Demand Side Management. It models the objective evaluation (based on maximum and minimum quality limits) and subjective evaluation (based on t-norms, acceptance level, hierarchy and uncertainty) through the Generic Camargo Intrinsic Cost Index and its methodology. This new index covers all these cases, without the need for a specific mathematical development for each of them, nor the use of different intrinsic cost indices depending on the case, which simplifies the methodology. In this way, Intrinsic Cost has been applied with different fuzzy operators in a generic new index: Algebraic, Generic and Particular Hamacher and Einstein Product.

Fifthly, from the four aspects mentioned above, the curves associated with the attributes analysed and Generic Camargo Intrinsic Cost and the efficiency frontiers separating the feasible and non-feasible areas are obtained. This resulted in a new computational model of market equilibrium in which the equilibrium points and zones were searched for and the marginal cost of equilibrium was obtained, as summarised below. Then theoretical, practical, and economic contributions of this methodology to the Argentinean production chain were presented through the introduction of a new market equilibrium model (Section 3.4.).

The main theoretical contribution the present methodology are the follows.

- The improvements obtained from the original proposal of the intrinsic cost index to the latest improvement of the present proposal are developed and analysed from a mathematical point of view. The original proposal was complex to implement even with two attributes, resulting in a large equation that was difficult to use and analyse. Therefore, improvements were made based on the Ceteris Paribus clause and by means of Hamacher's t-norm product families this expression was generalised for the four types of fuzzy intersection t-norms analysed.
- In this way, a general expression of intrinsic costs in cases of fundamental uncertainty (General Camargo Intrinsic Cost) has been obtained, which takes into account the objective valuation (incremental costs from the limits of the fuzzy function), the subjective valuation (acceptance levels or fuzzy preference functions of the indices studied) and the priorities (exponential weightings) of these indices studied (investment costs and emissions) from the decision maker's case: 1) Supply Side Management and 2) Demand Side Management. This is in line with the life cycle analysis carried out on the material and fuel flows for the five sectors studied: extraction, processing, manufacturing, construction and transport. In this way, the results obtained verified that the variants of the Camargo General Intrinsic Cost Index take into account the relative evaluation of the hierarchy (exponential weights) and the ranking according to the AHP, the objective evaluation of the non-monetisable attribute (differential cost) and its subjective evaluation (hierarchy of the decision maker). Through the novel improvements made to the proposed intrinsic cost index and the cases analysed, its relationship with economic theories has been demonstrated by obtaining the optimal (Particle Swarm Optimisation) supply and demand curves and a model for obtaining and analysing market equilibrium, which will be extended in the future. This is a novel way of incorporating fundamental uncertainty, objective and subjective economic evaluation of non-monetisable indices and priority criteria, for the three types of t-norm intersection of product t-norm. Future work would deepen this analysis and proposal, including current t-norms and potential state-of-the-art advances.

The main results of this methodology and its analysis are the follows (see Figure 3 to Figure 12).

- Graphically (see Figure 8, Figure 9 and Figure 11), it has been demonstrated that the slope of the Generic Camargo Intrinsic Cost curve is positive and therefore any increase in emissions will be penalised (demand side management) or incentivised (supply side management). It was observed that the slope of the generic Camargo intrinsic cost was inversely related to the factor p, which defines the family of t-norm Hamacher products. That is, when $p = 0$ (particular Hamacher's product), these intrinsic cost curves have high slopes, while for high values of p ($p = 2$ or Einstein's product), the intrinsic cost curve (Generic Camargo Intrinsic Cost) has low slopes. This is useful to know when defining the externality; if low values of p are chosen, the amount of the penalty or subsidy may vary significantly, and vice versa for high values of p. These t-norms can be used to adjust the slope of the supply-side management and demand-side management curves, as well as the slope of the intrinsic cost. In this way, the requirement for the penalty cost of emissions (carbon price) can be regulated.

- The graph of the investment costs and emissions (attributes) was examined in relation to the Generic Camargo Intrinsic Costs. The life cycle analysis yielded an emission interval of $\left[0.30;0.39\right]\frac{\text{Ton CO}_2\text{ EQ}}{\text{Mwh}}$ and the investment costs yielded an emission interval of $\left[1.5;5\right]\frac{\text{USD}}{\text{Mwh}}$, the limits of which correspond to the fuzzy limits (see Figure 1 and Figure 3 to Figure 12). The results were logical in both cases (Argentinean production chain and society).
- The interval for the Generic Camargo Intrinsic Cost (GCIC) was $\left[0;500\right]\frac{USD}{Ton\ CO_2\ EQ}$, reaching $500\frac{USD}{Ton\ CO_2\ EQ}$ in extreme solutions, while historical carbon bond prices fall within the interval $\left[20;100\right]\frac{USD}{Ton\ CO_2\ EQ}$ (see Figure 8, Figure 9 and Figure 11). The prices obtained were therefore in line with international values. As a result, the values obtained in the indices were in line with national and international studies such as (Camargo, 2019, 2021, 2022a, b, 2023; Argentina, 2023; Spain, 2023; Camargo et al., 2023a, b). The intrinsic cost proved to be a reliable indicator of energy efficiency and sustainability, as it includes both economic costs and environmental costs or benefits (CO_2 emissions). By improving the mathematical equation of the intrinsic cost index, this work has succeeded in extending the definition to other types of t-norms (continuous and differentiable).
- This model allowed for the determination of the break-even point of estuary prospects, based on supply and demand curves. Moreover, it demonstrated that the efficiency frontier of intrinsic costs was associated with marginal (opportunity) costs. This introduction laid the groundwork for future models enabling objective and subjective valuation of both positive and negative externalities, subject to fundamental uncertainty, in line with the efficiency frontier. The significance of these implications was emphasised, highlighting the practical implications of the present proposal. Additionally, the analysis was enhanced by examining the exponential weights variations and their implications graphically, further contributing to the comprehensive understanding of the proposed theoretical development. Overall, this study compiled and improved upon existing knowledge, yielding promising and insightful results.
- In this way, feasible and infeasible zones delimited by maximum and minimum desired efficiency curves were obtained with equilibrium zones and break-even points. Then, a break-even point (see Figure 12a) of $0.31\frac{Ton\ CO2\ EQ}{MWh}$ and $2.5\frac{USD}{MWh}$ was obtained and the break-even zone obtained where Emissions (U_i) were between $\left[0.3;0.31\right]\frac{Ton\ CO2\ EQ}{MWh}$ and the Investment Cost (U_j) were between $\left[1.6;3.4\right]\frac{USD}{MWh}$ (see Figure 12b). Through the newly established market equilibrium model, additional equilibrium points were obtained for the established priority criteria (exponential weights) based on the hierarchy of cases analysed. These points delineated a range of potential outcomes, similar to producer and consumer surplus in economics, illustrating the applicability of the current proposal to market equilibrium analysis.
- When this analysis was carried out, an interesting result was obtained (see Figure 9 and Figure 11). Firstly, increasing and decreasing intrinsic cost curves were obtained (see Equation 29 and Equation 30), and secondly, the boundary between the feasible and non-feasible zones formed a concave ‘U’. This was reminiscent of the marginal cost curve, which has the same characteristic, and therefore showed that the intrinsic cost effectively modelled the marginal cost of the variable being analysed. The curve had a minimum at $0.36\frac{Ton\ CO_2\ EQ}{MWh}$ and $50\frac{\text{USD}}{\text{Ton CO}_2\text{ EQ}}$, which was in line with current carbon credit prices, although not feasible according to the analysis in the following section. All solutions that were in the infeasible range (determined by the set of curves for each exponential weight that contained it) would have required less than the expected efficiency ($tp_{boundary}$), as explained in Section 3.4.2.
- The full model has been validated and calibrated using publicly available reports, and all these results calculated by Particle Swarm Optimisation are logical, consistent and correct with the data collected, and they are in line with the costs determined by the Argentinean Chamber of Renewable Energy and the Ministry of Energy and international values (Argentina, 2023; Spain, 2023).

The main practical and economical contribution of this methodology and its analysis are the follows.

- Given the exhaustive nature and broad scope of the comparison with various T-norms, it was considered impractical to fully develop it within this paper and therefore warranted further exploration in future studies. The results showed that fuzzy decision theory could delineate a zone and an equilibrium point depending on the hierarchy of the decision maker. This represented a significant advance in computable general equilibrium models, allowing the development of models that infer supply and demand equilibrium points from exponential weight (and vice versa). In particular, this could be achieved with variables with or without associated markets and with known or unknown prices, which was a notable advantage.

- In this way, it was then established that, from the theory of fuzzy decision making, it is possible to obtain an equilibrium zone and an equilibrium point that depend on the hierarchy of the decision maker. All these solutions are the infeasible region (defined by the set of curves for each exponential weight that make up the infeasible region) that require the minimum or maximum expected efficiency. This was a major contribution to general computational equilibrium theory, as it allowed the development of models that allowed the equilibrium point of supply and demand to be obtained from exponential weights (and vice versa). It is called Computable General Equilibrium (CGE) model. More importantly, this would be done with variables with or without an associated market, with known or unknown price, which is a great advantage. In future work, this analysis and proposal will be carried out and deepened with the current t-norms (and another possible state of the art).
- The results are therefore coherent, logical, satisfactory and promising and they can be applied to other areas of the productive sector, using other indices of interest. Therefore, this model can be perfectly used for economic models of the production chain, and this has the advantage of taking into account the fundamental uncertainty, since it works with fuzzy logic, and therefore it is a good proposal given the difficulty of modelling uncertainty in economic models. Future economic works will present and deepen the present proposal in this respect.

This work makes a significant contribution to the Argentinean production chain, providing tools for improving its efficiency and contributing to future models for analysing and predicting changes in its behaviour. Thus, this methodology has the advantage of being flexible (applicable to any problem), relatively easy to apply to complex problems, and incorporating metaheuristics and hierarchisation for multi-objective optimisation with fundamental uncertainty. In order to explore novel incentive and penalty mechanisms to improve energy efficiency and the carbon trading market, it was necessary to assess the suitability of the tools compared in this paper. Future work will further compare this research and apply mechanisms based on the Computable General Equilibrium (CGE) model of the Argentine supply chain and this methodology, with the aim of knowing the market price of carbon in the supply chain for each and every t-norm. In addition, the procedure to determine the price of energy investment projects based on these proposed indices will be applied. Mechanisms will also be implemented to calculate the economic amount of the penalty or subsidy for the externality produced. Innovative environmental valuation mechanisms have been developed in this work. With this carbon price, the regulator will provide an economic incentive for the company to make the relevant investments and to continuously search for energy efficiency. This is important in order to reach a compromise between the possibilities of the production chain and the needs of the users, based on the valuation of externalities. It will facilitate the development of carbon credit valuation mechanisms that take into account the specific environmental challenges, level of development, social inequalities, needs and hierarchy of each country.

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