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Research Article

Cost at Risk (CaR): a Methodology for Costing under Uncertainty

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Abstract

Paper aims: This paper proposes Cost at Risk (CaR), a concept and a methodology that allows the computation of the risk of cost estimations within a costing system by means of the Monte Carlo Simulation, considering a predefined level of confidence and considering the worst expected result in terms of cost in a certain period.

Originality: Traditionally, researchers and practitioners have been focused on deterministic costing models without recognizing and managing cost uncertainty. The proposed methodology is based on five steps that go from the determination of the parameters that generate uncertainty to the estimation of the risk.

Research method: A Design Science Research (DSR) approach was followed based on mathematical modeling and computer simulation.

Main findings: The model was applied to the imaging area of a hospital allowing to identify and quantify the risk of the most relevant costs and therefore, supporting the design and implementation of both operational and strategic decisions.

Implications for theory and practice: The main contribution is the inclusion in costing systems of the uncertainty inherent in the estimation of costs, particularly in complex environments.

Keywords

Costing Systems. Activity Based Costing. Risk management. Uncertainty. Monte Carlo Simulation. Cost at Risk.

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1. Introduction

Most companies are interested in determining costs more accurately in order to support decision making and firm's strategy as well as to improve the quality of their efficiency indicators which can positively affect products' profitability (Myrodia et al., 2017). Such models should reflect business and production processes and the firm's cost structure as accurate as possible. However, nowadays, organizations' internal and external circumstances change rapidly. Indeed, the complexity associated with the system's operation exacerbates cost unpredictability (Rodríguez et al., 2022). In this context, where endogenous and exogenous variables vary, cost behavior might be not fully understood if old-fashion costing models are used.

Indeed, the data used in costing systems is uncertain due to a heavy reliance on parameter estimation. This primarily exists because gathering the necessary information to compute product costs is an expensive process and a very time-consuming task. Furthermore, in many cases, business and production processes are highly varied and difficult to standardize. Thus, since costing systems data are historically based and often estimated, the true values of the data are uncertain, and the input data are likely to be inaccurate.

The need to consider the uncertainty in planning decisions dating back to the models of functional planning, where resources to be used in the future had been allocated based on current data and future projections.

Given that, the true value of each data parameter may never be known. Thus, it is important to acknowledge and manage the uncertainty within cost estimation and costing systems. This will allow the system user to acknowledge that the system results are not certain and potentially improve decisions affected by the system output by accounting for the inherent risk (Afonso & Jiménez, 2016; Durán & Durán, 2018; Ostadi et al., 2019).

Several methods have been designed for handling uncertainty through mathematical models. The Monte Carlo simulation and fuzzy methods can be highlighted among such methods (Díaz et al., 2022). Particularly, some authors have approached the subject of uncertainty in costing systems using either the Monte Carlo Simulation or Fuzzy methods (Durán & Afonso, 2021; Durán et al., 2019; Jiménez et al., 2015; Kropivšek et al., 2021; Ostadi et al., 2019; Rivero & Emblemsvåg, 2007).

The quantification of cost variability will allow answering questions such as: What is the cost risk of contracts and bids? Which variables, products, processes, or services are the cause of greater variability and therefore should be subject to greater control? Or, how to define measures and design actions of risk mitigation and evaluate the impact of such interventions?

It is important to consider uncertainty in costing systems however, even approaches based on Activity Based Costing (ABC) which are known as more advanced and complex than traditional methods, do not consider the uncertainty inherent in resources, activities, processes, cost drivers and cost objects. Traditionally, researchers and practitioners have been focused on the use of deterministic costing models without acknowledge and manage conveniently cost uncertainty. This paper presents and discusses a methodology for measuring the risk within costing systems, called Cost at Risk (CaR).

CaR can be defined as the maximum expected cost for a product or service for a future period, given a certain level of confidence. It considers the worst expected outcome in terms of costs in a certain period, for a pre-defined confidence level. Process and product variability can have a considerable impact on budgeting processes and consequently in cost management effectiveness. In this context, the general idea of CaR is the identification of the variables that generate cost variability and to model statistically those variables allowing the quantification of the level of risk associated to each cost element. Cost models that include uncertainty are fundamental for decision making and are at the highest level of maturity (Total Cost Management Division, 2017).

The methodology proposed in this paper was applied in the healthcare context, in which there are countless elements that induce variability in the cost of processes and services, such as morbidity, heterogeneity of patients, heterogeneity of medical treatments, among others. For example, such variability makes the cost of hospital services vary from one patient to another.

This is of significant importance because, in a hospital, the program contract or budget is produced considering services and processes which have inherent variability reflected in the variability in costs. Therefore, the contract will have variability over time considering such environment of high uncertainty what is, in general, poorly accommodated in the production and monitoring of the budget.

This paper is organized as follows. A literature review shows the importance of uncertainty in costing which has been included through tools such as fuzzy logic and Monte Carlo simulation. Then, a methodology for estimating the risk in costing is proposed which starts from a deterministic model complemented with Monte Carlo Simulation to include uncertainty. The model was applied in the imaging area of a hospital since the cost of each patient is characterized by a high uncertainty and depends on several factors such as the patient's health status and the clinical procedure to be followed. Finally, the main conclusions and opportunities for further research are presented at the end of the paper.

2. Literature review

2.1. Risk measurement

If a problem can be modelled, it allows the decision maker to be better informed not only on the behavior of a system but also allows him to estimate possible future behaviors of that system and to design the corresponding action measures (Abed et al., 2015). In the process of modeling a phenomenon, the decision maker must decide whether to include the uncertainty associated with it. In certain cases, an approach based on a deterministic model could be sufficient, considering that more detailed information would not generate significant differences in terms of decision making. On the other hand, when uncertainty is significant and seems to be decisive in understanding the behavior of the phenomenon, such uncertainty must be included in the model. When that happens, the decision maker must choose an appropriate method to model such uncertainty (Zimmermann, 2000).

It should be noted that there is not a definitive definition of uncertainty. Although, there is some consensus in very specific areas (Grenyer et al., 2021). For example, in the financial sector when people talk about risk and uncertainty, risk is associated with the knowledge of a probability distribution of the occurrence of a phenomenon, and uncertainty with the ignorance on such distribution (Yazdi et al., 2019). But, it is generally recognized that uncertainty implies that in a given situation a decision maker does not have information of a quantitative or qualitative nature that allows him/her to fully understand the behavior of a phenomenon (Zimmermann, 2000).

The uncertainty can be derived from several conditions related to a phenomenon, such as, the lack and complexity of information, human errors, the existence of a few similar phenomena, and other exogenous factors to the phenomenon such as economic, political and social changes, among others (Hazır & Ulusoy, 2020; Oehmen et al., 2020). According to Nachtmann & Needy (2003), uncertainty is related to the possibility of error that derives from not having complete information about a phenomenon and the environment where it is inserted. Thus, it can be said that there is a direct relationship between the complexity of the information and the degree of uncertainty of a phenomenon.

Among the techniques used to capture the uncertainty we can highlight the following: the Decision Trees, Markov modelling, Fuzzy Logic, Discrete Event Simulation (DES) and Monte Carlo Simulation (MCS) (Jiang et al., 2023; Jiao et al., 2022; Liu et al., 2022; Nguyen et al., 2022; Wang et al., 2022).

The choice of the appropriate method for modeling uncertainty depends on the type of problem that we want to model. Among the methods used, we have interval math, probability theories, fuzzy set theory, approximate set theory, theory of the evidence, etc., which, in general, propose mathematical models to treat and measure uncertainty. Interval mathematics is the simplest way to represent uncertainty. It requires the definition of a lower and upper limit. Such representation does not consider the probabilities of occurrence of events (Mo et al., 2019). The theory of fuzzy logic was introduced in the 1960s to quantify the lack of precision and uncertainty. The main objective in fuzzy sets is to quantify inaccurate information (Soto et al., 2020).

A well-known approach to work with uncertain environments is the Monte Carlo simulation. In the simplest case, the method randomly samples a universe of outcomes and take the fraction of the results that fall within a given range, as the sample size increases the results converge to a certain value. The central limit theorem provides information on the probable magnitude of the error in the estimate after a finite number of draws (Glasserman, 2004; Wu & Zhang, 2022). The Monte Carlo simulation method can be included in the experimental branch of mathematics that deals with experiments with random numbers based on statistical probabilities (Kroese et al., 2014). Essentially in the Monte Carlo simulation, probability distributions functions of the variables associated with the occurrence of a phenomenon are defined and then through statistics it is about measuring and possible behavior of the phenomenon, what is commonly called a risk analysis or a measurement of uncertainty (Afonso & Jiménez, 2016; Esmalifalak et al., 2015).

About MCS, it is an appropriate tool to obtain numerical solutions to problems difficult to solve analytically and currently, computational tools allow modeling complex situations, offering quickly answers for decision making. Usually there is a logical sequence of steps to model an economic evaluation problem, ranging from problem identification, model building, and sensitivity analysis (Jang et al., 2022).

2.2. Risk and uncertainty in costing systems

There are significant sources of uncertainty which impact the estimation of costs and their allocation to cost objects, such sources may exist in the design of the product, in its production and even in its commercialization. The increasing complexity and dynamic nature of engineering systems drives an inherently high level of uncertainty. The uncertainty is reduced when there is more information about the product (Grenyer et al., 2021).

Another source of uncertainty in the estimation of costs derives from the fact that the information to perform the estimation can be expensive or time-consuming and in some cases the processes are very varied and difficult to standardize. Since the bases used for cost estimation are usually historical or predetermined, it is likely that the input data for cost estimation is inaccurate. Therefore, it is important to recognize and manage uncertainty within cost management systems. This will allow the decision maker not only to recognize the possible results of the cost estimation but also to generate the respective intervention measures (Durán & Afonso, 2021; Esmalifalak et al., 2015; Gupta & Maranas, 2003; Nachtmann & Needy, 2003).

To quantify this uncertainty, several methods have been used. One of the first approaches that have been used to measure and understand the uncertainty of costs and measure the level of risk is sensitivity analysis. The sensitivity analysis offers a first look for the decision maker on the stage about what would happen if the most relevant inputs change considering a more optimistic and pessimistic situation or in the expected scenario. However, the approach has some limitations, one of the most relevant is that it is difficult to use when there are multiple variables that explain the behavior of a phenomenon and these variables are also correlated. To address this limitation, tools based on probabilistic methods have been designed which provide better information than sensitivity analysis (Datta & Roy, 2010).

One of the most studied and used in the estimation of costs is the activity-based method (ABC), one of the limitations of this method is that in its conception it is not incorporated the existence of uncertainty in the elements of cost (Ostadi et al., 2019).

Cost management relies on the principle that costs should be known to use such information for decision making (Henri et al., 2016). Activity-Based Costing (ABC) can be used to calculate more accurately and distribute better indirect costs which is an increasingly important component of the total cost. An activity-based costing system is based on the premise that activities consume resources and cost objects (e.g., products and services) consume activities. The development of effective costing systems requires a good design of processes and activities (Calvi et al., 2019). Resources supply and support activities and, in general terms, can be classified in human resources, equipment, informatics resources, materials and others. Resources' variability will have a direct impact in the number of calculations that must be performed to calculate the cost associated with products or services (Fei & Isa, 2010; Lueg & Storgaard, 2017).

For the distribution of indirect costs, proper cost drivers should be selected to relate resources to activities and the later to products. The most common drivers are volume-based, such as the number of machine hours, number of man-hours, number of products, number of lots, number of setups, etc. Default rates for associating resource consumption with activities and between activities and the associated products must be computed. Cost rates are obtained dividing total costs by the quantity of the selected driver (Ostadi et al., 2019).

By focusing on activities, ABC offers, among others, the following advantages: it identifies the activities that do not add value, identifies expensive or inefficient processes, facilitates continuous improvement, and offer information for the reduction of costs (Gosselin, 2006; Parker, 2016). The ABC can be used also to understand the cost implications of additional internal activities that might be required to meet different customers' specific requests (Magnacca & Giannetti, 2023).

However, ABC data are often estimated due to cost and time constraints, which leads to inherent imprecisions and uncertainty (Nachtmann & Needy, 2001; Ostadi et al., 2019). Indeed, in general, the data used to develop costing systems are uncertain due to a heavy reliance on parameter estimation. This primarily exists because gathering the necessary information to generate product costs is an expensive process and because, in some cases, the processes are varied and difficult to standardize. Since data are typically historically based and often estimated, the true values of such data are uncertain, and the input data are likely to be inaccurate. The need to consider the uncertainty in planning decisions dating back to the models of functional planning, where resources for the future are allocated based on current data and future projections. Given that the true value of each data parameter may never be known, it is important to acknowledge and handle the uncertainty within the costing system. This will allow the system user to have a better understanding of the behaviour of the system and its inherent risk (Rinaldi et al., 2022).

To mitigate the limitations of deterministic costing systems, some authors have proposed extended ABC models which incorporate uncertainty in the cost variables (Afonso & Jiménez, 2016; Durán et al., 2019; Jahan-Shahi et al., 1999; Jiménez & Afonso, 2016; Ostadi et al., 2019; Sarokolaei et al., 2013).

Jahan-Shahi et al. (1999) applied fuzzy sets and probability distribution methods to address uncertainty in cost estimation. Nachtmann & Needy (2001) developed a methodology based on the theory of fuzzy sets to manage the imprecision of estimation and uncertainty in ABC systems. Nachtmann & Needy (2003) have studied and compared different methods to manage uncertainty in ABC systems. The methods studied were interval math, Monte Carlo simulation (with triangularly distributed input parameters and normally distributed input parameters) and fuzzy set theory. The authors concluded that Monte Carlo simulation and the fuzzy set theory are better than interval math to manage uncertainty in such costing systems. Ostadi et al. (2019) used fuzzy logic to compensate for the lack of reliable and definitive data in the Time Driven-Activity based Costing (TDABC). Durán et al. (2019) use a methodology Fuzzy ABC for estimated the costs (including uncertainty) of the surface treatment and water treatment activities can be appropriately allocated to the products that generate those activities and that waste.

3. Materials and methods

A Design Science Research (DSR) approach was followed in this research project. In DSR the goals of academic research are very pragmatic. Although generalization is not one of the strengths of this method, the results can be extrapolated to projects with similar characteristics. DSR generally requires the creation of an artifact, theory, model or drawing to present, understand, and / or enhance a reality. Often, DSR creates innovative artifacts to solve complex, real-world problems and generate design knowledge (Akoka et al., 2022; Baskerville et al., 2015; Hevner et al., 2004).

Following the DSR approach, the CaR methodology is proposed through a series of defined steps, and then applied as a real-world case study. Cost-at-Risk (CaR) is analogous to Value-at-Risk (VaR) which is a statistical measure of market risk that estimates the maximum loss that could register a portfolio in a time span, considering a certain level of confidence. Nevertheless, unlike VaR which considers the left tail of the distribution of output (i.e., lower income), CaR highlights the right part of the distribution (i.e., higher costs). The CaR methodology follows the five steps procedure of VaR (RiskMetrics Group, 1999): Metric specification, Exposure mapping, Scenario generation, Valuation, and Risk measuring computation (Jorion, 2000; Liu & Ralescu, 2017).

3.1. Metric specification

Firstly, it is necessary to define the financial measure to which the risk will be measured, usually earnings or cash, specifying a time horizon and a certain level of confidence. In this case, the marginal gain in the calculation of the risk (or quantification of the variability) must be analyzed with the marginal cost of obtaining it. It should be defined in which products or services this analysis has value for decision makers, which can be prioritized using for example a Pareto analysis. But, in many cases, such prioritization can be not so obvious due to the existence of multiple attributes asking for the use of some technique of multi-attribute prioritization such as the Maximin, the Maximax, the elimination and choice expressing reality (ELECTRE) (Govindan & Jepsen, 2016) , the analytic hierarchy process (AHP) (Saaty, 2016), among others, to determine which products worth measuring risk (Seppälä et al., 2001).

A costing model is needed at this step. Several methodologies for cost estimation can be used, one of the most has been activity-based costing (ABC) which is based on the logic that activities consume resources, and products consume activities. A good knowledge of the process is essential for the design of correct costing models in general and of ABC, in particular. It includes identifying products and other relevant cost objects (e.g., services) as well as the processes and the activities. Commonly, this is achieved by performing an analysis and mapping of the processes of the organization.

For the allocation of costs, appropriate drivers should be selected to relate resources to activities and the later to products. The most common drivers are volume-based, such as the number of machine hours, number of man-hours, number of products, number of lots, number of setups, etc.

The predetermined rates relate the consumption of resources by each activity and the consumption of activities by each product. These rates are obtained by dividing the total costs by the total quantities of the selected driver.

For the estimation of costs, the following ABC model was used, considering the following parameters.

$CR := \left(cr_{ij}\right)_{nx1}$ = Cost of resources used

$AR := \left(ar_{ij}\right)_{mxn}$ = Relation between resources and activities, where $\sum_{i=1}^{m} ar_{ij} = 1 \ \forall j$

$CA := AR \cdot CR = \left(ca_{ij}\right)_{mx1}$ = Cost of each activity

$PA := \left(pa_{ij}\right)_{oxm}$ = Relation between activities and products, where $\sum_{i=1}^{o} pa_{ij} = 1 \ \forall j$

$CP := PA \cdot CA = \left(cp_{ij}\right)_{ox1}$ = Cost of each product or service

This model allows to compute the cost of products in a deterministic way. It is important to note that due to the large amount of information that a production process can have, once the computation of product costs, a Pareto analysis must be performed to identify products, activities, and resources worth continuing to be studied and which will be taken into consideration in the model with uncertainty. This allows us to reduce the information needed to support the decision making.

3.2. Exposure mapping

The second step for the quantification of the CaR is defined as exposure mapping, i.e., for the metric defined in step 1, the sources of uncertainty or risk must be identified. This step is fundamental for the inclusion of uncertainty in the cost estimation model. For that, it is necessary to identify probability distributions which allow to understand the variability of the process.

In the case of an ABC system, it is possible to identify uncertainty related to the variability in how activities consume resources for each product or service provided, or variability in how products consume activities. If there is sufficient historical data, several methodologies and statistical tests of fit can be used to determine the empirical distribution of the data.

For example, the Anderson-Darling (AD) test is a non-parametric test on whether the data in a sample come from a specific distribution (Anderson & Darling, 1954; Coronel-Brizio & Hernández-Montoya, 2010). The Kolmogorov-Smirnov (K-S) test is also a non-parametric test that determines the goodness of fit of two probability distributions to each other. In case of wanting to verify the normality of a distribution (Massey Junior, 1951), it is the Lilliefors test which brings some improvements with respect to the one of Kolmogorov-Smirnov (Lilliefors, 1967). The Shapiro-Wilk test is one of the most powerful tests for the normality contrast, particularly for small samples (n<50) (Shapiro & Wilk, 1965).

When there is little information to find the empirical distribution, a theoretical distribution that fits the studied phenomenon can be used. Among the theoretical distributions associated to real-life phenomena, the most used are the binomial distribution, Poisson distribution, uniform distribution, normal distribution, and exponential distribution, the first two are discrete and the last three continuous. These distributions can be obtained from similar studies, by analogy or from the opinion of experts as illustrated by studies such as Page et al. (2014) in which expert opinion is used to construct probability distributions functions of the variable sources of uncertainty.

3.3. Scenario generation

Step three is related to the generation of scenarios. For each measure identified in step 2, there is generally an extensive list of scenarios or possible values that the variable should take. Historical Simulations, Delta-Normal Approach and the Monte Carlo Simulation can be used for this generation. Through the simulation analysis it is possible to estimate the expected cost of the selected cost objects and their corresponding probability distributions.

To build the model with uncertainty, it is assumed that $\mathrm{ar_{ij}}$, $\mathrm{p_{ki}}$ and $\mathrm{tr_j}$ are uncertain parameters. A sample $\mathrm{ar_{ij}^e}$, $\mathrm{p_{ki}^e}$ and $\mathrm{tr_j^e}$ is generated for each input parameter $\mathrm{ar_{ij}}$,$\mathrm{p_{ki}}$ and $\mathrm{tr_j}$, using their probability density function (PDF) Which is derived from the analysis performed in step 2.

It is important to note that some variables may be interrelated. Such relationships can be determined statistically by calculating the coefficient of Spearman between each of the variables. The Spearman correlation coefficient ρ measures the strength of the relationship between ordinal variables which, instead of the observed value uses the order of the variables. Thus, this coefficient is not sensitive to asymmetries in the distribution, or the presence of outliers, thus not requiring that the data come from two normal populations. For a sample of size n, the n raw scores X_i, Y_i are converted to ranks x_i, y_i, and the spearman correlation coefficient (ρ) is defined as:

$$\rho = 1 - \frac{6 * \sum d_i^2}{n\left(n^2 - 1\right)} \tag{1}$$

Where, $d_i = x_i - y_i$, is the difference between ranks.

3.4. Valuation

Step four is called valuation. In this case, the expected result must be calculated for the result or output defined in the first step, the value of $\mathrm{cp_k^e}$ is the outcome variable, which is calculated considering:

$$\mathrm{cp_k^e} = \mathrm{f}\left(\mathrm{ar_{ij}}, \mathrm{p_{ki}}, \mathrm{tr_j}\right) \tag{2}$$

Where:

$$ar_{\mathrm{ijt}}^{\mathrm{e}} = \left[ar_{\mathrm{ij1}}^{\mathrm{e}}, ar_{\mathrm{ij2}}^{\mathrm{e}} \ldots, \mathrm{ar_{mns}^e}\right] \tag{3}$$

$$\mathrm{p_{kit}^e} = \left[\mathrm{p_{ki1}^e}, \mathrm{p_{ki2}^e} \ldots\ldots, \mathrm{p_{oms}^e}\right] \tag{4}$$

$$tr_{jt}^{e} = \left[tr_{j1}^{e}, tr_{j2}^{e} \ldots\ldots, tr_{ns}^{e} \right] \quad (5)$$

The procedure is repeated for s number of iterations. Finally, the outcomes are analyzed using statistic criteria, histograms, confidence intervals, among other statistics. When calculating the probability distribution associated with the cost of each product, it is possible to estimate the probability of a specific cost value, which could be the maximum or minimum expected cost for a given product. It is known that each of the estimated output data has a probability of occurrence based on the random distribution used for the simulation. If the values are ranked from the highest to the lowest and confidence level is chosen for the cost of 95% when the cumulative probability reaches this value, we will find the Cost at Risk – CaR –. Figure 1 shows graphically the determination of CaR.

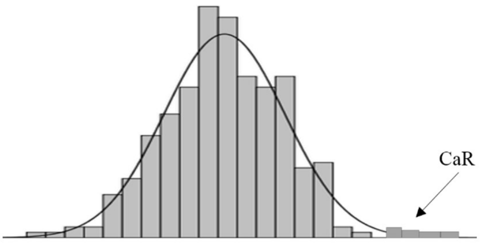

Figure 1. Cost at Risk.

CaR is important for policyholder decision makers and for identifying those products that deserve more attention, which products can most affect the profit, and which strategies can be used to mitigate or eliminate the risk level. Unlike VaR which considers the left tail of the distribution (i.e., lower income) CaR highlights the right tail of the distribution (i.e., higher costs), as shown in Figure 1.

3.5. Risk measuring computation

Possible probabilities distribution functions (PDF) of the variables that affect the cost can be considered namely, Uniform, Triangular, Lognormal, Weibull, Binomial, Poisson, among others. These PDFs can be used with expert information and be defined *a priori* when not enough statistical information is available. However, as soon as possible it should be collected information on the random variables. The distributions can be adjusted to the empirical distributions using a goodness-of-fit test, using the tests Anderson-Darling, Kolmogorov-Smirnov, and Chi-Square. Among the reasons for adjusting the distributions, we may highlight the establishment of correlations between the input variables, which can have a great impact on the forecast results.

4. Application

4.1. Context

The CaR methodology was applied in the imaging service of a Portuguese hospital. It is a medium-high complexity hospital of the public National Health Service network that annually performs more than one million six hundred thousand external consultations, nine hundred thousand emergency episodes, ninety-four thousand surgeries and more than eighteen thousand births. The hospital offers services in the areas of pathological anatomy, anesthesiology, general surgery, cardiology, intensive care unit, cardiovascular surgery, dermatology, gastroenterology, gynecology and obstetrics, imaging, internal medicine, neurology, oncology, among others.

The imaging service which is a medical specialty that is based on obtaining images for diagnosis and therapy of various types of pathologies, using different types of radiation such as X-ray, ultrasound, and radio frequency treatments. The imaging department provides services to other departments of the hospital.

Receiving patients from different departments and specialties creates an implicit variability in the service that is provided, as this will depend not only on the service that comes (i.e., an emergency patient may require more care and different services than an outpatient) but also on the pathologies the patient may possess what is not anticipated by health care providers in general and hospitals in particular. In this research project it was important to study the different processes and the differences among the diverse types of exams performed in the Imaging services.

The variability not only generates uncertainty in the type of treatment to be performed, but also can generate a risk of complying with the expected cost of a given treatment. That is why it is not only necessary to quantify such variability but also to measure its impact on the fulfillment of the contract and the budgeting process.

This service performs over 150,000 exams a year, but a careful analysis showed that it is possible to aggregate them into homogeneous groups of exams. These homogeneous groups are the following: Support to other services, Gastrointestinal imaging, Urologic imaging, Ultrasound, Mammography, X-ray, interventional/vascular radiology, non-contrast-enhanced MRI (magnetic resonance imaging), Contrast-enhanced MRI, Non-contrast-enhanced computed tomography, and Contrast-enhanced computed tomography.

The process in the imaging service was divided into five activities namely, scheduling the patient, receiving the patient, making the exam, processing the exam, and preparing the report.

Furthermore, resources were classified into human resources, equipment, informatics resources, materials, and others. About human resources, they were considered the physicians and all the staff at the imaging area including administrative and production elements. In terms of informatics resources, the various information systems responsible to deliver or receive information were considered. In this case, they were found a total of 8 different information systems, which in most cases have no communication among each other. In terms of equipment, they were considered all the machines and equipment (e.g., Scanner, Ultrasonography machine, Interventional radiology) used to produce the exams. Finally, among the materials found we must highlight direct materials (medicines and surgical items) and indirect materials (related to maintenance equipment, administrative materials, hotel equipment, treatment equipment, electro-medical equipment). Medicines are responsible for over 70% of total material costs.

Once the activities, resources, cost drivers and cost objects were identified we were able to design the costing model and apply the CaR calculation methodology.

4.2. Deterministic model

The results of the deterministic model allowed the identification of the cost objects in which a risk analysis would make more sense due to its large contribution to the total cost. Then, it is shown how the variability of the problem was incorporated from the deterministic model, to follow the estimation of the CaR and finally the sensitivity analysis and its impact on decision making. One of the most complex problems to deal in the imaging service is related to the consumption of materials, since it cannot be standardized to each patient. It basically depends on the patient's age and other personal characteristics. By applying the proposed model, they were found interesting results for the distribution of materials and consequently for the cost of the exams by patient.

Table 1 shows the unit cost for each exam in terms of materials. Values are displayed in percentage terms to maintain the confidentiality of the data. These results come from the deterministic cost model that was built using Microsoft Excel spreadsheets for data processing.

Table 1. Material Cost by Type of Exam.

Type of exam	Medicines	Treatment Material	Electro-medicine Material	Other Materials	Total Cost
Contrast-enhanced computed tomography	48.173%	0.460%	0.459%	0.297%	49.390%
Contrast-enhanced MRI	23.448%	0.334%	0.333%	0.213%	24.327%
X-ray	0.396%	0.059%	2.581%	4.389%	7.425%
Interventional/vascular radiology	0.046%	3.824%	0.000%	0.224%	4.095%
Non-contrast-enhanced computed tomography	0.066%	1.323%	1.538%	0.995%	3.922%
Support to other services	0.512%	1.616%	0.507%	0.332%	2.966%
Mammography	0.000%	0.000%	2.491%	0.035%	2.526%
Urologic imaging	1.313%	0.374%	0.000%	0.586%	2.273%
Gastrointestinal imaging	0.874%	0.660%	0.000%	0.015%	1.549%
Ultrasound	0.427%	0.014%	0.102%	0.966%	1.509%
Non-contrast-enhanced MRI	0.001%	0.006%	0.007%	0.004%	0.018%
Total	75.255%	8.671%	8.018%	8.056%	100.000%

About 90% of the costs are concentrated in materials processing, medicine and medical imaging, and these costs are related to two types of examination, which are contrast-enhanced MRI and contrast-enhanced computed tomography. It was determined that the largest proportion of these costs are due to two types of materials used (of about a total of 300) in the imaging area which represent about 65% of the total cost of materials consumed in the service. The application of these materials depends on the age of the patient, his/her weight, and other personal parameters and attributes.

In this same line of reasoning, the existing variability of materials cost was also analyzed. Table 2 shows that medicines preset the greatest variability.

Table 2. Statistics by Type of Material.

Statistics	Type of Material			
	Treatment Material	Medicines	Electro-medicine Material	Other Materials
Min	0.00%	0.00%	0.00%	0.00%
Max	3.82%	48.17%	2.58%	4.39%
Mean	0.79%	6.84%	0.73%	0.73%
Standard Deviation	1.1426%	15.36%	1.00%	1.26%

The results shown in Table 2 allow to identify medicines as one of the sources of uncertainty that must be studied and included in the risk analysis and that should be subject to greater control by the decision makers. On the other hand, Table 3 shows the analysis of the variability made by type of exam. greater variability in the materials related to Contrast-enhanced mri and Contrast-enhanced computed tomography, being the latter where the greatest variability occurs. This information is valuable because it allows prioritizing the way the efforts should be addressed and in the case of having to choose where to begin the risk analysis, we should consider not only the contribution to the total cost, but also the level of internal variability of resources.

Table 3. Statistics for the Relationship between the Type of Material and the Exams.

Type of exam	Statistics			
	Min	Max	Mean	Standard Deviation
Support to other services	0.00%	1.62%	0.34%	0.75%
Gastrointestinal imaging	0.00%	0.87%	0.17%	0.50%
Ultrasound	0.01%	0.97%	0.22%	0.41%
Mammography	0.00%	2.49%	0.28%	1.28%
X-ray	0.03%	4.39%	1.05%	2.73%
Interventional/vascular radiology	0.00%	3.82%	0.46%	1.96%
Contrast-enhanced mri	0.00%	23.45%	2.71%	13.41%
Non-contrast-enhanced mri	0.00%	0.01%	0.00%	0.34%
Contrast-enhanced computed tomography	0.00%	48.17%	5.50%	27.77%
Non-contrast-enhanced computed tomography	0.01%	1.54%	0.48%	0.94%
Urologic imaging	0.00%	1.31%	0.25%	0.61%

Thus, it was evident that these two exams are the ones that had the greatest impact on materials costs and consequently on the services provided. Therefore, they were considered as critical variables to be monitored. Table 2 and Table 3 shown the variability associated to both the resources and the cost objects, allowing to define the inputs that must be modeled for a good analysis of risk and uncertainty.

Due to the lack of consumption records by patient, we proceeded to obtain the information about the possible behaviors that the consumption of medicines considering the knowledge of clinical staff. After several meetings, it was defined an empirical distribution based on the following hypothesis: it is possible to make improvements in processes that contribute to improve overall efficiency by 5%, however given the increasing complexity of patients, deterioration of equipment and other external factors, consumption could increase by 15%. These assumptions allowed to construct a triangular distribution for each of the variables identified as critical or with greater uncertainty and to evolve from the deterministic to the stochastic cost model.

4.3. Stochastic model

Triangular distributions have been used in numerous studies carried out in hospitals in which it is intended to analyze uncertainty and there is little information to determine an empirical distribution. The studies carried out by: Bhattacharjee & Ray (2016), Mestre et al. (2014), Zimlichman et al. (2013) are good examples. One of the reasons for the use of such distribution is that it only depends on three parameters. Triangular distributions are continuous probability distribution functions with lower limit (a), upper limit (b) and mode (c), where $a < b$ and $a \leq c \leq b$. The limits a, b and c are easy to understand by those who are responsible for prestart or manage the services.

A total of 20,000 scenarios using the Crystal Ball software were simulated. The results are presented in Table 4.

Table 4. Simulation Statistics.

	Variability Contrast-Enhanced Computed Tomography	Variability Contrast-Enhanced MRI	Variability of Total Cost
Trials	20,000	20,000	20,000
Base Case	0.00%	0.00%	0.00%
Mean	2.60%	2.92%	2.15%
Median	2.26%	2.56%	1.91%
Mode	---	---	---
Standard Deviation	3.29%	3.78%	2.39%
Variance	0.11%	0.14%	0.06%
Skewness	0.3002	0.2710	0.3725
Kurtosis	2.31	2.27	2.53
Coefficient of Variation	1.26	1.29	1.11
Minimum	- 4.27%	- 5.09%	- 3.06%
Maximum	11.04%	12.49%	9.49%
Range Width	15.31%	17.57%	12.55%
Mean Std. Error	0.02%	0.03%	0.02%

Considering that the values are presented in percentage; it is to say the CaR with 95% confidence was estimated as the 95th percentile of the distribution of the percentage difference of the total cost. To do this, and after analyzing, triangular distributions were defined for the consumption of drugs, the correlations between the critical variables were included and the total cost for each type of procedure was defined as the output variable. This estimation of the correlations can be considered a weakness, since it is based on the experience of the clinical staff instead of using patients' direct information which is not available.

Table 4 shows the simulation statistics and due to the conditions predicted above it is possible to expect an average variation of the total cost of 1.91%. It can also be seen that the range of variability is asymmetric, having a positive asymmetry which in terms of cost indicates that there is more probability of having growth than a decrease in costs.

This asymmetry can partly be explained by the increase in patient complexity that is expected and has been perceived by clinical staff. In turn, it influences growth in the expected cost for successive periods also because the lack of incentives of the clinical staff to improve the efficiency of the processes. In terms of contracting, these results have several implications. Namely, monitoring and control activities should be carried out to reduce the consumption of these resources without affecting the quality of the service and this can be done using consumption records by patients and procedures.

In addition, these results allow to predict what can happen to the budget compliance in these services, and to classify the situations as normal or abnormal, which is a valuable information for decision makers.

With respect to CaR related to the contrast-enhanced computed tomography, the contrast-enhanced MRI and the Total cost of Materials they were produced several analysis which can be evidenced in Figure 2, Figure 3 and Figure 4 respectively.

The figures show the variation in percentage of these variables once the simulation process was performed. It can be said that with the 95% confidence the CaR for the Contrast-Enhanced Computed Tomography, the Contrast-Enhanced MRI and the total cost of material is 8.46%, 9.46% and 6.48% respectively. Recognizing this variability is important because to the extent that most patients are above the average cost, the probability of non-compliance with the general budget will be greater, which is why intervention measures must be generated to reduce said variability.

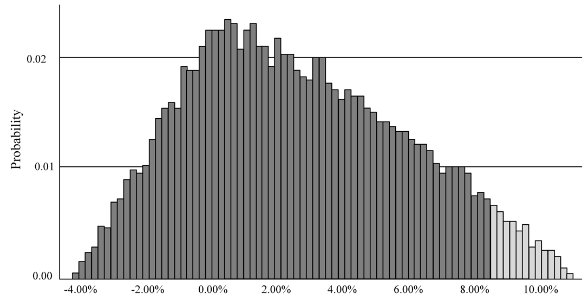

Figure 2. Variability of the Total Cost of the Contrast-Enhanced Computed Tomography.

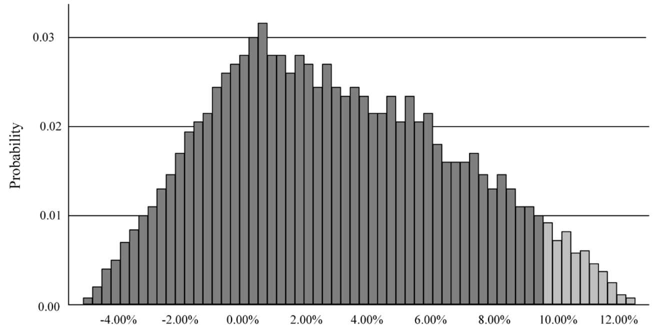

Figure 3. Variability of the Total Cost of the Contrast-Enhanced MRI.

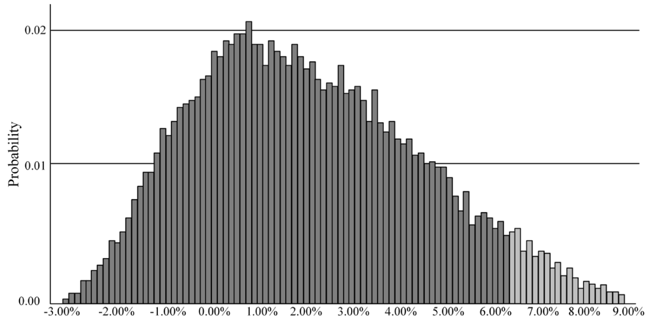

Figure 4. Variability of the Total Cost of Materials.

The CaR methodology allows not only to identify extreme values in the cost due to variability, but it can also help to define ranges for the cost that allow to define when the cost is "under control" - i.e., when it is within the range, and when intervention measures should be generated. In order to identify which of the variables has the higher impact in the total cost and therefore must be object of monitoring and control, it was performed a sensitivity analysis which is presented in Figure 5.

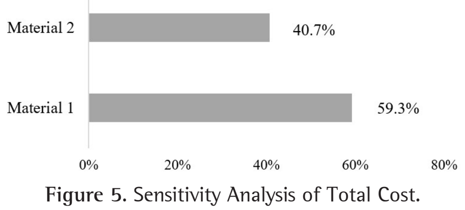

Figure 5. Sensitivity Analysis of Total Cost.

This analysis showed that the material that most contributes to the variability of total direct costs is Material 1. Thus, this material was considered critical, and the administration took action to improve its utilization to meet the budget for this type of procedures. One of the decisions was to make complete records of the use of the material by each patient. These records allow a better management from the clinical perspective and help in the adjustment of the CaR according to the evolution of the condition of the patients treated in the Hospital.

4.4. Discussion

The understanding of the behavior of costs is a task that is increasingly important given the variability in markets, prices, methods of work, among other aspects, which induce variability in the cost of final products and services. This variability can affect the efficiency of the organization by the influence of endogenous and exogenous factors; respectively, some controllable and other not controllable by the managers. Consequently, a better understanding of the process variability contributes to a better control of costs.

Particularly, cost management in hospitals is very important as hospitals and generally public services have fixed operating budgets and the only variable that can be controlled to improve their financial performance is the production cost. In addition, improving hospitals' compliance with budgets contributes to improved overall compliance in financial terms of the national health system, and non-compliance may also have negative impacts that may affect not only health financing but also the quality of the service provided. Furthermore, hospitals are important because they absorb much of the budget dedicated to health within any country (Hellowell, 2013).

The model proposed here to deal with uncertainty and risk suggests that it is possible that the efficiency of a hospital does not only be function of internal processes and how costs are managed and allocated, but also that also it is significantly influenced by the type of patients served by the hospital. In this case, unit materials cost may vary significantly depending on the patient's own characteristics.

It is interesting that in hospitals there is an implicit variability due to the type of patients that are attended. Thus, this variability can affect the efficiency of the hospital by an exogenous element not controllable by hospital managers and administrators.

The existence of variability generates risks in the fulfillment of the objectives namely, the fulfillment of what has been established in the contract or on what has been budgeted. That is why risk measurement in costing becomes a key tool for decision making in organizations whose processes and services are subject to variability such as the case of hospitals. Thus, the most developed cost management models must consider the risk management associated to resources, activities, and cost objects CII (Total Cost Management Division, 2017).

Two important aspects that should be considered for future applications are the following: Firstly, compare the results obtained with distributions defined a priori with the experts (triangular, normal, or uniform) with the results obtained from the empirical distributions of the data. The second aspect is related to the estimation of the correlations between the cost elements (materials, labor) and the execution times of each of the examinations, for this it is necessary to collect the information for each of the patients for each pathology.

To the extent that reality can be modeled and simulated, the marginal benefit of the information obtained will greatly outweigh the marginal costs of obtaining such information. These models take relevance in organizations to the extent that they serve to support decision making. For this reason, it is important that organizations understand the need to identify, quantify and manage the cost associated with their products and services. Furthermore, they must understand the importance of including the uncertainty associated to resources and activities that influence the cost of the products and services. This will allow a proper management of the associated risk.

Comparing our model with the models proposed by Nachtmann & Needy (2001), Namazi (2009) and Sarokolaei et al. (2013), we can highlight some advantages, in the first place the fact of presenting a cost model that can be adapted to several situations, taking into account the correlations between variables, the number of scenarios generated and the sensitivity analysis. Another advantage is the fact of being structured following the logic of widely used models such as the VaR estimation proposed by Group (RiskMetrics Group, 1999) Jorion (2000) and Liu & Ralescu (2017).

5. Conclusions

Managing the inherent uncertainty and inaccuracy associated to cost parameters provides the user with information that can be useful for many proposes including those related to product, activities and process decisions, bidding and budgeting activities, make-or-buy analyses, among others. Indeed, costing models that may deal adequately with uncertain are particularly beneficial in several contexts namely, when the company is operating in an uncertain environment, the input data is inaccurate or inadequate, when there is a lack of confidence in the accuracy of the estimated input data, when the level of overhead costs is large enough to affect relevant decisions, etc.

Measuring the uncertainty associated with costing systems will turn cost management more effective through for example a proper budgeting and a better financial risk management. Indeed, the identification of potential losses related to excessive or insufficient costs may permit to design contingency plans or propose intervention measures or strategies of risk mitigation.

Within an organization, risk management is focused on achieving high performance objectives, as well as ensuring the stability and solvency of the company. There are different methodologies that allow companies to analyze the risk to which companies are exposed and that can be used to design strategies towards the mitigation of those risks. These methodologies range from the application of classical statistical models to computer simulations. All this to generate decision-making tools that are consistent with the company's policies and strategies without affecting the expected returns and guaranteeing the fulfillment of what has been budgeted or contracted for a certain period.

In this sense, it was proposed here a methodology for the calculation of CaR structured in two interrelated phases. Firstly, a deterministic model is constructed for determining the costs associated to the relevant cost objects. Secondly, a probabilistic model using Monte Carlo simulation is used to include uncertainty in the cost analysis. The Monte Carlo simulation model proposed here may allow to some extent to predict the risk associated to the variability in costs and support the necessary steps which should be taken to better manage such risk, whether from the point of view of processes optimization and of cost management.

The available computational tools and resources makes the Monte Carlo simulation a useful tool not only for risk measurement but also for organizational decision making at reasonable time and with a marginal cost lower than the marginal benefit of obtaining information related to the risk associated with costs, the fulfillment of contracts, budget objectives, etc.

One aspect that stands out in the CaR methodology is its flexibility to be adapted and applied in different contexts and complementing other models. Thus, further work can be done to develop and extend the proposed methodology. Namely, to compare the results of using different techniques such as fuzzy techniques versus the Monte Carlo method. Different costing systems can also be used. For example, the idle capacity is important and impacts significantly in costs, and TDABC models are particularly appropriated to deal with the cost of unused capacity. On the other hand, the CaR methodology can be applied in different business processes and production environments for both services and products. Integrated computational tools can be also considered to facilitate the access to the inputs and the generation of outputs. Finally, further work can be to done using the CaR methodology in optimization models, where the cost is usually assumed as an exogenous parameter. Nevertheless, as evidenced in this paper, costs can be seen as variables whose complexity depends on the problem that is being solved.

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